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Travelling along the Silk Road: A new interpretation of Ptolemy's coordinates

Irina Tupikova,¹ Matthias Schemmel,² Klaus Geus³

Introduction

The interpretation of the coordinates given by Ptolemy in the description of *Scythia on this side of the Imaon* and *Serike* on the map Asia 8 (*Geogr.* 6.15.1–6.16.8) has always been one of the most challenging tasks of ancient geography. First, the comparison of the Ptolemaic coordinates with their modern counterparts is not unambiguous: the longitudes of the locations are linked to different “prime” or “zero” meridians in the Ptolemaic and the modern coordinate systems. The positioning of Ptolemy’s Prime Meridian – the Islands of the Blest (*Insulae Fortunatae*), i.e. the Canaries – was not flawless: the islands lie along the same meridian, far too south from their actual latitude and the longitudinal difference between them and the next point on the European coast, Cabo de San Vicente, attains 2° 30’ instead of 4°– 9°. Second, further to the east the longitudinal difference between the modern and Ptolemaic coordinates for identified locations increases steadily, thus illuminating the notorious fact of the excessive distortion of Ptolemy’s map in the east-west direction.⁴ Usually, modern scholars explain this error as an attempt on Ptolemy’s part to fit the extension of the *oikoumene* to 180° for the sake of symmetry or for better application of his cartographic projections. We shall argue here that this extension can be explained without assuming Ptolemy to have adjusted the whole data set, which he had at his disposal, to a prescribed doctrine which would not be based on the material itself.

The characteristics of the longitudinal extension of the Ptolemaic maps can be explained and illustrated by the following procedure. Let us consider one important location A, e.g. *Rome*, *Alexandria* or *Cologne*, which could serve, with all probability, as starting or reference point for local mapping in Ptolemy’s cartographical approach. This reference position A has a certain

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⁴ For a recent investigation of the longitudinal distortions in Ptolemy’s *Geography* see Shcheglov 2014.

longitudinal value, λ_A reckoned from the Greenwich meridian used as a prime meridian in the modern cartographical system. Assuming that the longitude of this location A was precisely known in Antiquity and attained in Ptolemaic coordinates the value of λ^P_A , then, the Ptolemaic longitude λ_G of the Greenwich Meridian can be calculated as $\lambda_G = \lambda^P_A - \lambda_A$.

By subtracting λ_G from the Ptolemaic longitude for some other location B in the vicinity of the reference point A, one obtains the “Greenwich” longitude of B as $\lambda_B = \lambda^P_B - \lambda_G$.

When repeating this procedure for different locations in the vicinity of A we obtain a map where all modern coordinates are distorted in a peculiar way. This pattern shows that nearly all locations lying to the east of A display an eastern digression from their actual positions and, vice versa, the locations to the west of A will show mainly a western digression.⁵ In fact, every single map extends locally relative to each place used as a starting point of mapping in Ptolemy’s original cartographical procedure.

The reason for this expansion has already been formulated by Henry F. Tozer in his *A History of Ancient Geography*⁶ as follows:

With regard to the circumference of the Earth Ptolemy followed Marinus in accepting Posidonius’ erroneous estimate of 180,000 *stadia*, which fell short of the reality by one-sixth. It resulted from this that, as he adopted from Hipparchus the division of the equator and other great circles into 360 degrees, he made every degree only 500 *stadia* (50 geographical miles) instead of 600 *stadia* (60 geographical miles), which is the true computation. This mistake at once affected his calculation of distances on his map, for in consequence of it he overestimated them: thus, if he discovered from the authorities – itineraries or otherwise – that the interval between two places was 500 *stadia*, he would express it on his map by a degree, which in reality is 600 *stadia*; and then the estimate was made on a large scale, the error in excess became very great. This was especially felt when he came to deal with the second important question of general scientific geography, that of the length of the habitable world, because he greatly overestimated this relatively to the true circumference of the earth.

Leaving aside the question of the source of the “true” estimate for the Earth’s circumference mentioned by Tozer, one can only marvel why such a simple idea has not yet become a scientific paradigm in Ptolemy-related studies. Naturally, the majority of the source material which Ptolemy had at his disposal for global mapping of the known *oikoumene* were not tables with spherical co-

⁵ For a map identifying Ptolemy’s longitude of Alexandria with its actual longitude, see Stückelberger 2009: 267, Fig. 2.

⁶ Tozer 1897: 341–342.

ordinates, latitudes and longitudes (probably not even earlier geographical works like that of Eratosthenes or Strabo), but measured, estimated or inferred terrestrial distances (sometimes expressed in different units) which he had to convert into arc measures. In such recalculations, the adopted size of the Earth – or, equivalently, a certain number of *stadia* assigned to one degree – is of primary importance. Let us recall that some indications⁷ point to the fact that Ptolemy initially used in his *Almagest* the Eratosthenian estimation of the circumference of the Earth as 252,000 stades⁸ which would attain a metrical value of 700 stadia for one degree.

The mathematical consequences for mapping distances gained at a sphere of a certain size at a sphere of another size were first discussed in Tupikova/Geus 2013. For example, the excessive distortion of Ptolemy's map along the east-west direction can now be understood as the inevitable result of his erroneously adopted size of the Earth in combination with attempts to preserve the latitudes of some locations gained through astronomical observations.⁹ The other consequences are a) the impossibility to determine globally the position of Ptolemy's prime meridian in the geographical coordinate system, b) the mutual displacement of the local maps linked to different reference points and c) the far too northern (or far too southern) positioning of locations lying approximately on the same meridian as the reference point of mapping.

An algorithm was proposed for recalculations of spherical coordinates given on a sphere with the circumference of 180,000 units (Ptolemy's number in his *Geography*) to a sphere with the circumference of 252,000 units (Eratosthenes' estimate of the circumference of the Earth).¹⁰ The results of our recalculation have shown that if Ptolemy had adopted Eratosthenes' figure, most of his positions would have had coordinates which match their modern counterparts remarkably well. As a consequence, the very high precision of Eratosthenes' result for the circumference of the Earth was confirmed, as well as the equivalence of the length of a *stadion* used by both scholars, at least for the Mediterranean world.¹¹

An attempt to apply the same method to more distant regions such as *Skythia* and *Serike* may raise some questions. The common opinion is that the sketchy description of these regions resulted from erroneous conversions by

⁷ See especially Schnabel 1930: 214–250.

⁸ For the different values ascribed to Erasthenes, see Appendix 1.

⁹ Isaksen advances much evidence that Ptolemy used in his mapping also a scheme of preconceived hour intervals (Isaksen, in preparation).

¹⁰ Because spherical coordinates are dimensionless, the metrical value of a *stadion* is of no importance in recalculation.

¹¹ In Tupikova/Geus 2013 the data sets have been investigated especially for the maps Europe 2–4.

Ptolemy (or already by his predecessor, Marinus) of alien measurement units (e.g. *schoinoi*, *parasangs* or *li*) into stades. But, in fact, the problems of an erroneously adopted size of the Earth and of wrong conversions of distances expressed in hardly known units are *mathematically equivalent* and should be treated by the same formulae.¹²

Strictly speaking, the Ptolemaic coordinates need to be recalculated separately for every region where we can suspect a possible error in converting distances. Of course, to do this, we should know the “true” (in fact, not standardized) relations between a Greek *stadion* and other measuring units. Having completed such a recalculation, we would obtain a Ptolemaic map where the distances given in “alien” units were correctly transformed into his *stadia*, but the possible error in the circumference of the Earth (leading to $1^\circ = 500 \text{ stadia}$) is still preserved. This error could then be corrected in a second step of our recalculations. Fortunately, as our results suggest, in the case at hand the problem of the conversion of units can be largely neglected in the reconstruction of Ptolemy’s geographical data.

For the purpose of identifying locations, statistical methods can be of great help. In well-known regions, where correctly identified locations provide both the Ptolemaic coordinates and their modern counterparts, one can construct a system of relations between these sets of coordinates supplied with some unknown parameters.¹³ If the Ptolemaic set of coordinates is free from systematic errors, one is able to determine these parameters through minimization of deviations of the coordinates and can then apply the results to the not yet identified positions in order to find their modern equivalents. In the case, however, when the size of the Earth was erroneously adopted, this method can be used only with great caution. Not only will a systematic error creep into any set of the Ptolemaic coordinates; there exist also some specific cartographical problems discussed together with some examples in *Appendix 2*. To this set of problems belong the cases which we term “overmapping”, “inversion” and “doubling” – notorious and recurrent features in Ptolemy’s *Geography*, often leading to misinterpretations by modern scholars. Statistical analysis *per se* cannot explain them.

For the Ptolemaic map of *Scythia on this side of the Imaon*, where one is hardly able to find a couple of correctly identified locations, such statistical methods cannot be applied at all.

¹² The mathematical background of the problem is discussed in *Appendix 2*.

¹³ See, e.g., Kleineberg/Marx/Knobloch/Lelgemann 2010, Kleineberg/Marx/Lelgemann 2012, Marx/Kleineberg 2012.

The Route to the Tarim Basin

In all probability, the author of the *Geography* was drawing on data that had been assembled and transmitted since several centuries before and was combining these with more recent information he had at his disposal. The route from Hierapolis at the crossing of Euphrates towards *Scythia on this side of the Imaon* is described by Ptolemy in the chapters entitled “On the computations that Marinus improperly made or the longitudinal dimension of the *oikoumene*” (*Geogr.* 1.11) and “The revision of the longitudinal dimension of the known world on the basis of journeys by land” (*Geogr.* 1.12). That is, the primary interest of Ptolemy here concerns finding out the angular distance to the most remote part of the known world. We give these chapters in Berggren/Jones’ translation:¹⁴

11. The foregoing should have made it clear how far it would make sense to extend the latitudinal dimension of the *oikoumenē*. Marinus makes its longitudinal dimension bounded within two meridians that cut off fifteen hour-intervals. We think that he has also extended the eastern part of this dimension more than necessary, and that when a reasonable reduction has been applied here, too, the whole longitudinal extent does not amount quite to twelve hour-intervals, where we (like [Marinos]) set the Islands of the Blest at the westernmost limit, and the farthest parts, [namely] Sēra, Sinai, and Kattigara, at the eastern [limit]. For in the first place one should follow the numbers of stades, from place to place, set down by [Marinos] for the distance from the Islands of the Blest to the crossing of the Euphrates at Hierapolis (as if [the journey] were made along the parallel through Rhodes). [This is] both because it is continually being checked and because [Marinos] has manifestly taken into account the amount by which the greater distances ought to be corrected on account of diversions and variations in the itineraries. Furthermore, [he has taken into account] the fact that one degree (of such as the great circle is 360°) contains 500 stades on the surface of the earth - in accordance with the surface measurements that are generally agreed upon-while an arc similar to [one degree of the equator] on the parallel through Rhodes (that is, the parallel 36° from the equator) contains approximately 400 stades. (We may ignore, in such a rough determination, the slight excess over [400] that follows from the [exact] ratio of the parallels.) However, we reduce according to the appropriate correction both the distance from that crossing of the Euphrates to the Stone Tower, which amounts (according to him) to 876 schoinoi or 26,280 stades, and that from the Stone Tower to Sēra, the metropolis of the Sēres, a journey of seven months, or [according to Marinos] 36,200 stades reckoned on the same parallel [through Rhodes].

¹⁴ Berggren/Jones 2000.

For in the case of both journeys, [Marinos] has clearly not subtracted the excess resulting from diversions, and in the case of the second, he has fallen as well into the same inconsistency that he also fell into concerning the journey from the people of Garame to Agisymba. There he was compelled to subtract more than half from the number of stades added up over [a journey of] four months and fourteen days because the road journey could not have been uninterrupted over such a great time. Logically this ought also to have been the case with the seven months' journey, indeed, much more so than with the route from Garame. After all, that journey was made by the country's king, who had (as is reasonable to suppose) some considerable advance knowledge [of the route], and the weather was completely favourable. But the route from the Stone Tower to the Sēres is subject to bad storms (for according to Marinos' assumptions it falls on the parallels through the Hellespont and Byzantion) so that for this reason, too, there must have been numerous pauses in the journey. Moreover, it was because of the opportunity for commerce that [the route] came to be known. Marinos says that one Maes, also known as Titianus, a Macedonian and a merchant by family profession, recorded the distance measurements, though he did not traverse it himself but sent certain [others] to the Sēres. [Marinos] himself apparently did not trust merchants' reports: at least, he did not give assent to the account of Philemon, in which he has reported the longitudinal extent of the island of Hibernia [i.e., Ireland] from east to west as a twenty days' journey, because [Philemon] said that he heard it from merchants. For, [Marinos] says, these merchants do not concern themselves with finding out the truth, being occupied with their commerce; rather, they often exaggerate the distances out of boastfulness. But here also the circumstance that nothing else in the seven months' journey was deemed worthy of any record or report by the travellers reveals that the length of time is a fiction.

12. For these reasons, and also because the road is not on a single parallel (rather, the Stone Tower is near the parallel through Byzantion, and Sera is south of the parallel through the Hellespont), it would appear sensible here, too, to diminish the number of stades added up from the seven months' itinerary, namely 36,200, to less than half. Let it, however, be reduced just to half; for this rough determination, so that the distance in question will be reckoned as 18,100 stades, or $45\frac{1}{4}^{\circ}$. It would, after all, be absurd and unheard of; when reason dictates the same size of reduction for both the routes, to follow it in the case of the route from the people of Garame [to Agisymba] because the refutation was staring us in the face (namely the various animals in the country of Agisymba, which could not be moved outside their natural places), yet in the case of the route from the Stone Tower [to Sera], not to accept the logical consequence since such a refutation did not happen to be applicable there because the environment along the whole distance would be the same, whether [the distance] be greater or smaller just as

if someone were not to act rightly, [that is], in the manner appropriate to philosophy, so long as he was not about to be caught. The first interval, by which I mean the 876 *schoinoi* from the Euphrates to the Stone Tower, must be reduced, because of the diversions in the routes, to just 800 *schoinoi*, or 24,000 stades. For, granted that the total distance of the whole [route] may be believed because it has been measured in moderately sized parts that have been much travelled, nonetheless it is obvious even from Marinus' assumptions that it has numerous detours. It is true that the route from the crossing of the Euphrates at Hierapolis through Mesopotamia to the Tigris, and from thence through the Garamaioi in Assyria and Media to Ekbatana and the Caspian Gates, and to Hekatompylos in Parthia, can be situated near the parallel through Rhodes, since this parallel, according to [Marinos], too, is drawn through the countries mentioned. But the road from Hekatompylos to the city of Hyrkania must veer to the north, since the city of Hyrkania lies more or less between the parallel through Smyrna and the parallel through the Hellespont because the parallel through Smyrna is drawn right under the country of Hyrkania, while that through the Hellespont is drawn through the southern end of the Hyrkanian [Caspian] Sea, which is a little to the north of the city of the same name [i.e., Hyrkania]. Again, the road thence to Antiocheia Margiane through Areia inclines at first to the south, since Areia lies on the same parallel as the Caspian Gates, and then to the north, since Antiocheia is situated near the parallel through the Hellespont. Thence, the road to Baktra extends to the east, from there to the ascent of the range of the *Kōmēdai* [the road goes] to the north, and from this range to the gorge that follows upon the plains [it goes] to the south. For [Marinos] places the northern and the westernmost parts of the range, where the ascent is, on the parallel through Byzantion, and the southern and the eastern parts on the parallel through the Hellespont; this is why he says that [the route], though it leads pretty well straight east, tends to the Notos [south] wind. And apparently the fifty *schoinoi* from thence toward the Stone Tower incline to the north, for he says that as one ascends the gorge, the Stone Tower comes next, and from thence the mountains go off to the east and join up with the Imaon [range], which goes up from Palimbothra to the north. Thus when the 60° that correspond to the 24,000 stades have been added to the $45 \frac{1}{4}^\circ$ from the Stone Tower to the Sēres, the distance from the Euphrates to the Sēres along the parallel through Rhodes would be $105 \frac{1}{4}^\circ$. And according to Marinus, on the basis of the individual numbers of stades that he assumes, and reckoning as on the same parallel, the distance from the meridian through the Islands of the Blest to the Sacred Cape of Spain amounts to $2 \frac{1}{2}^\circ$; and that from thence to the mouth of the Baetis, and that from the Baetis to the Straits [of Herakles] and Calpe each amounts again to $2 \frac{1}{2}^\circ$. And, of the following [intervals], that from the Straits to Caralis in Sardinia amounts to 25° , that from Caralis to Lilybaeum in Sicily to $4 \frac{1}{2}^\circ$, that

from thence to Pachynus to 3° , and next that from Pachynus to Tainaros in Lakonia to 10° , that from thence to Rhodes to $8\frac{1}{4}^\circ$, that from Rhodes to Issos, $11\frac{1}{4}^\circ$, that from Issos to [Hierapolis on] the Euphrates to $2\frac{1}{2}^\circ$. Thus the sum for this distance is 72° , and for the whole longitudinal extent of the known earth, from the meridian of the Islands of the Blest to the Sēres, $177\frac{1}{4}^\circ$ in total.

To summarize, Ptolemy divided the route into three parts. For the first part, “one should follow the numbers of stades, from place to place, set down by [Marinus] for the Island of the Blest to the crossing of the Euphrates at *Hierapolis* (as if [the journey] were made along the parallel of Rhodes)”. This distance was adopted by Ptolemy without correction because “it is continually been checked and because [Marinus] has manifestly taken into account the amount by which the greater distances ought to be corrected on account of diversions and variations in itineraries”. An unclear point here is whether Marinus suggested the *Insulae Fortunatae* in fact to lie at the latitude of Rhodes (36°) instead of latitudes between $10^\circ 30'$ and 16° which Ptolemy gives later in his catalogue for this archipelago.¹⁵ If not, Marinus would have had to perform a complicated trigonometric recalculation of the known distance to the islands to obtain the distance measured along the parallel of Rhodes. Such complex mathematics, which at that time was based on tables of chords, can hardly be ascribed to Marinus himself: from quotations of Ptolemy one has rather the impression that Marinus proceeded with an estimated length of degree along the standard parallels of Rhodes, Byzantium or Alexandria.

The second part of the route, the distance from the crossing of the Euphrates to the *Stone Tower*, which amounted according to Marinus to 876 *schoinoi* or 26,280 stades, was reduced by Ptolemy to 800 *schoinoi* because “the total distance of the whole [route] may be believed because it has been measured in moderately sized parts that have been much travelled, nonetheless it is obvious even from Marinus’ assumptions that it has numerous detours”. In the third part, from the Stone Tower to *Sera*, the metropolis of the Seres, the distance is given as a journey of seven months, or according to Marinus of 36,200 *stadia*. Applying arguments which sound rather phony (such as that the route from the Stone Tower to the Seres is subject to bad storms), Ptolemy reduces this distance to 18,100 *stadia*. We will show further that the reason for such a radical reduction may be attributed to the very practice of cartographical work – configuration of a global map by adjusting

¹⁵ The *Insulae Fortunatae* in the Ptolemaic catalogue lie too much to the south. In our mind this is also a consequence of the erroneously adopted size of the Earth and their position can easily be corrected with our formulae (see Tupikova 2014).

local maps. The position of Sera, e.g., could be aligned with the position of Palimbothra by the old routes connecting China and India. This revision of Marinus' distances is given schematically in Fig. 1.

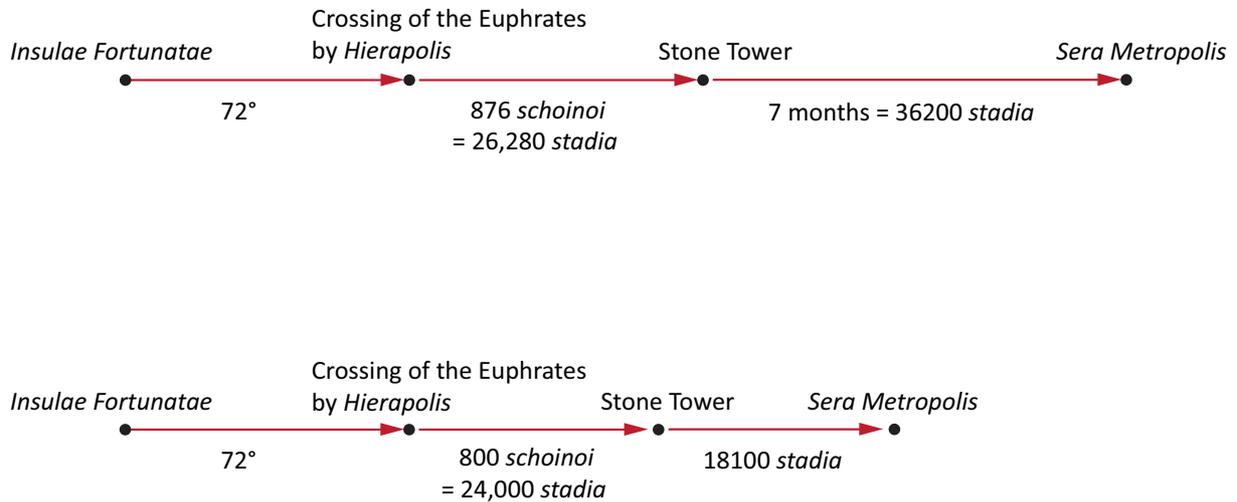


Fig. 1. Ptolemy's reduction of Marinus' distances.

The important locations mentioned along the route are as follows (latitude; longitude):

1. *Hierapolis*/Membidj (*Geogr.* 5.15.13): 36°15'; 71°15'
2. *Ekbatana*/Hamadan (*Geogr.* 6.2.14): 37°45'; 88°
3. *Caspian Gates* (*Geogr.* 6.2.7): 37°; 94°
4. *Hekatonpylos* (*Geogr.* 6.5.2): 37°50'; 96°
5. *Hyrkania* (*Geogr.* 6.9.7): 40°; 98°30'
6. *Antiocheia Margiane*/Merv (*Geogr.* 6.10.4): 40°40'; 106°
7. *Baktra*/Mazar-e Sharif =Balkh (*Geogr.* 6.11.9): 41°; 116°
8. Ascent to the range of *Komedai* (*Geogr.* 6.13.2): 43°; 125°
9. Gorge of *Komedai* (*Geogr.* 6.13.2): 39°; 130°
10. *Stone Tower* (*Geogr.* 6.13.2): 43°; 135°
11. *Hormeterion* (ὄρμητέριον) (*Geogr.* 6.13.1): 43°; 140°.

Two other important locations connected with the route are:

12. *Marakanda*/Samarkand (*Geogr.* 6.13.2): 39°15'; 112°
13. *Alexandreia Eschate*/Khujand (*Geogr.* 6.12.6): 41°; 122°.

On the following maps we will denote the modern position of locations mentioned in our text with green, the Ptolemaic positions with red and the position recalculated relative to different reference locations with yellow circles. The positions of the mountain passes are marked by stars.

Since the crossing of the Euphrates at *Hierapolis* is mentioned as an important starting point for the route towards *Scythia on this side of the Imaon*, we first of all give here the Ptolemaic positions in the region reduced for the meridian of Greenwich defined relative to this location, i.e. by adjusting the Ptolemaic longitude of this point with the meridian of Membidj. The locations in Fig. 2 are numbered according to the list given above.

The already mentioned stretch of the Ptolemaic maps is visible at first glance: e.g., *Baktra* lies at the longitude of Kuqa, in the middle of the Tarim basin. The characteristic error in latitudes of such important locations in this region like *Baktra* or *Antiocheia Margiane* was inherited by Ptolemy via Marinus from, possibly, Eratosthenes: they were supposed to lie at the latitude of the *Hellespont*. As a consequence, the latitudes of the locations connected with these cities – *Ekbatana*, *Caspian Gates* and the not yet reliably identified *Hekatonpylos* and *Hyrkania* – show increasing latitudes. In contrast, the latitude of *Hierapolis* as well as the one of *Marakanda* were very well estimated.

The method of recalculating positions to the Eratosthenian size of the Earth was discussed first in Tupikova/Geus 2013. The main idea is to obtain a solution with the methods of spherical trigonometry. A location's position is recalculated relative to a selected reference point from the spherical triangle with vertices at the reference point, North Pole and the position itself. Of course, the position of a reference point itself may be erroneous; nevertheless, the distance and the direction to the location whose position we have to recalculate were in fact known and measured from the *real* position of the reference location. It seems therefore reasonable to transfer the direction and the distance between both locations to the exact position of the reference point and then proceed to recalculate the coordinates for the "Eratosthenian" Earth. In the following text every reference to "recalculated positions" is meant as the coordinates calculated from the original Ptolemaic coordinates relative to a selected reference point according to this scheme. The position of the Greenwich Meridian will be adjusted to this reference point.

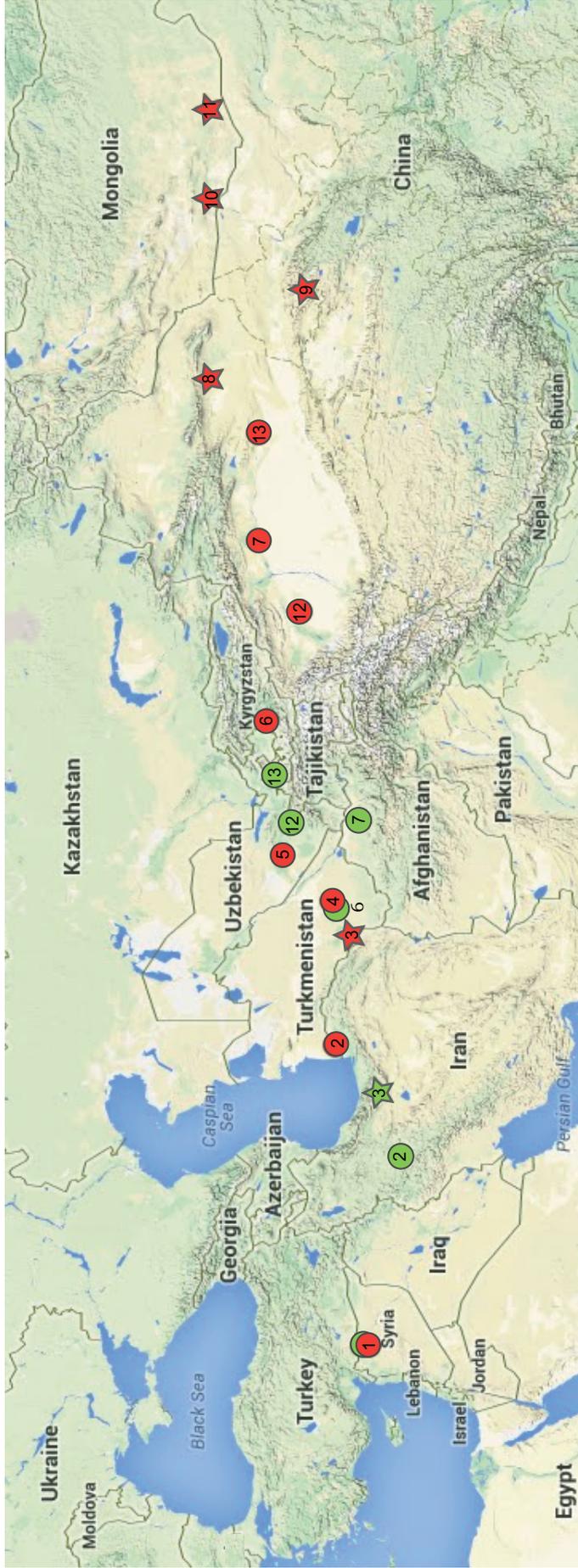


Fig. 2. Ptolemaic positions (in red) vs. identified positions (in green) along the route towards *Scythia on this side of the Imaon and Serike*. The position of the Greenwich Meridian is defined by identifying the Ptolemaic longitude of *Hierapolis* with the longitude of Membidj.

Map data ©2015 Basarsoft, Google, INEGI, Inav/Geosistemas SRL, ORION-ME.

We would like to emphasize that our method does not (strictly speaking) *improve* the positions given in Ptolemaic catalogue. With our method we just *obtain* the positions *which Ptolemy himself would have calculated* if he had chosen the circumference of the Earth as equalling to 252,000 *stadia*. Our procedure still retains all the original Ptolemaic errors caused mainly by the limited geographical knowledge and methods of that time. It allows modern scholars, e.g., to evaluate the accuracy of Ptolemy's original data.

The positions recalculated relative to two reference points – Alexandria and the crossing of Euphrates at *Hierapolis/Membidj* – are given in Figs. 3 and 4.¹⁶ The recalculated coordinates now slide towards their proper longitudinal positions – that is, the basic error for the overextension of the Ptolemaic world map is eliminated. The characteristic errors in the latitudes of *Antiocheia Margiane* and *Baktra* (40° 40' and 41° instead of 37° 40' and 36° 45', respectively) are still preserved and spoil the positions of the locations adjusted to them: the positions of *Ekbatana*, *Caspian Gates*, *Hekatonpylos* (often identified as Shahrud) and (possibly) *Hyrkania* show a small but systematic overestimation of their latitudes. This overestimation is cut down, however, if we choose Alexandria as reference point. Probably, Ptolemy linked them not with the position of *Hierapolis* but directly with Alexandria. In contrast to that, the position of *Marakanda* seems to be mapped by the known distance to *Membidj*; after recalculation, it exhibits perfectly the correct position of Samarkand. The peculiar position of *Marakanda* which lost its status as the metropolis of Sogdiana at the time of the *itineraria* (i.e. Maes Titianos¹⁷) used by Marinus and which is even misaligned to Baktria (instead of Sogdiana) on the Ptolemaic map, is in fact the only uncorrupted position in this region. Its accurately known position relative to Hierapolis can be attributed to Alexander's "foot-measurers" (βηματισται). The precision of *Marakanda's* coordinates is not obvious in the original Ptolemaic coordinates; its erroneous placing in Baktria even forced J. Markwart to conclude:

Hieraus geht wohl hervor, dass die im System des Ptolemaios so seltsame Lage von Marakanda (39° 15' Br.), welche der wahren Breite der Stadt auffallend nahe kommt, nicht etwa auf einer wirklichen, vom Kartographen benutzten astronomischen Breitenbestimmung beruht, derzuliebe derselbe die Stadt in einer Provinz verrückt hätte, welcher sie

¹⁶ For recalculation we have used the formulae of *Case 2* (see Tupikova/Geus 2013), i.e. we have assumed that the directions and the length of the routes were known and involved in Ptolemaic mapping procedure. For three positions, *Antiocheia Margiane*, *Baktra* (because Ptolemy refers directly to their latitudes) and *Marakanda* (whose latitude coincides perfectly with its real value) the coordinates were recalculated with the formulae of *Case 1* (known latitudes and length of the routes).

¹⁷ For a recent evaluation of Maes Titianos, see Dan 2013.

nicht angehörte, wie Mannert, *Geographie der Griechen und Römer* IV 459 (vgl. Forbiger, *Handbuch der alten Geogr.* II 562 A. 72) für möglich hält. Denn sonst hätte Marinus gewiss eher die Lage von Baktra und Antiocheia Margiane nach der von Marakanda korrigiert, als den Nachrichten über die serische Handelstrasse zuliebe Marakanda von Sogdiana getrennt und diese Provinz weit nach oben verrückt.¹⁸

Our recalculation of the Ptolemaic coordinates to the Eratosthenian size of the Earth strongly suggest that the precise latitudinal value of *Marakanda* coupled with a perfect longitudinal value indeed results from a real astronomical observation of the latitude in combination with highly precise terrestrials measurements, possibly carried out by the *bematists* of Alexander the Great.

The second interesting case in point concerns the position of *Alexandria Eschate*. In contrast to other locations, it shows a larger longitudinal displacement relative to its actual position. In our opinion, this city served as a kind of “linking point” between two different local maps. It seems to be connected with position of the mouth of River *Rha*/Volga at the Caspian Sea which is itself mapped relative to Rome. The information about the distance to the mouth of the *Rha* river Ptolemy probably gained from older *itineraria*.¹⁹

¹⁸ Markwart 1938: 140–141.

¹⁹ Rapin 1998/2001: 201–205.

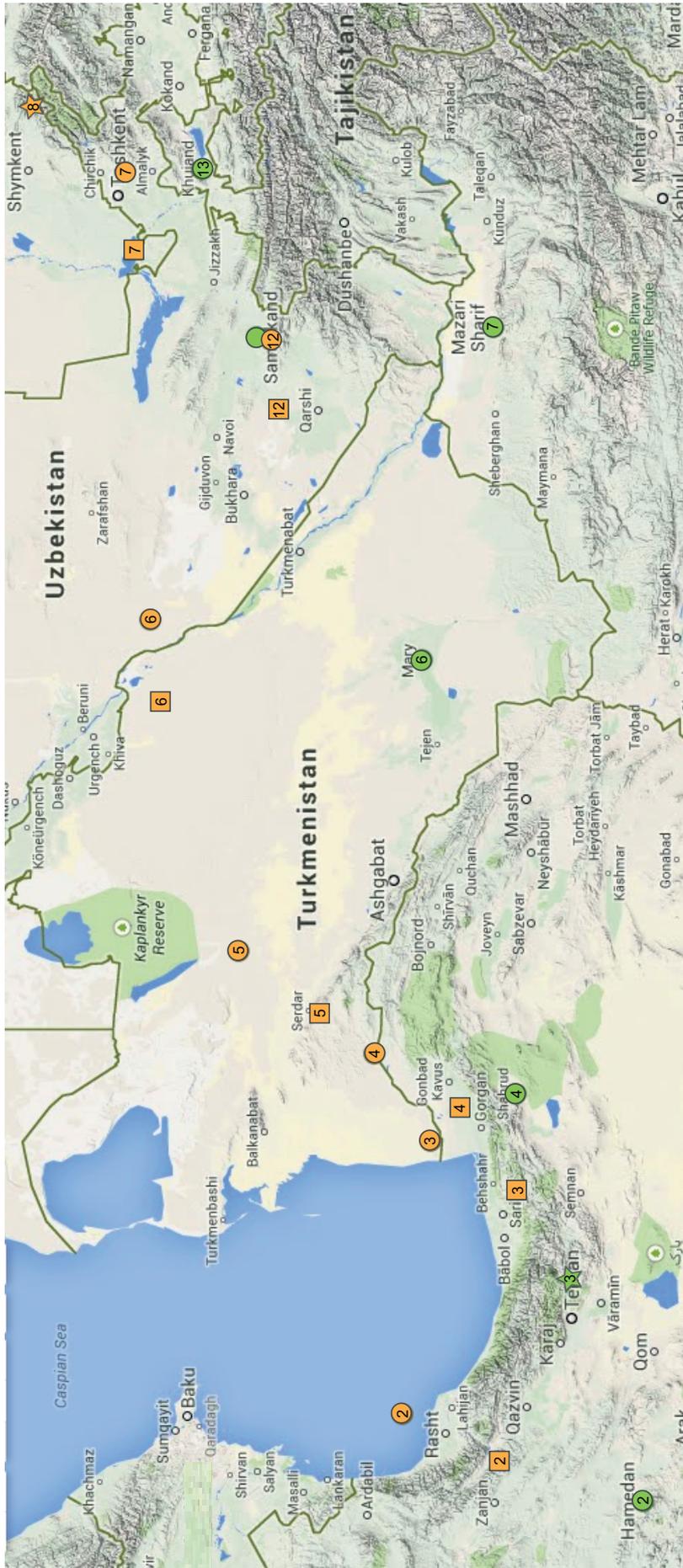


Fig. 3. Positions recalculated relative to Alexandria (yellow squares) and relative to Hierapolis/Membidj (yellow circles).

Map data ©2015 AutoNavi, Basarsoft, Google.

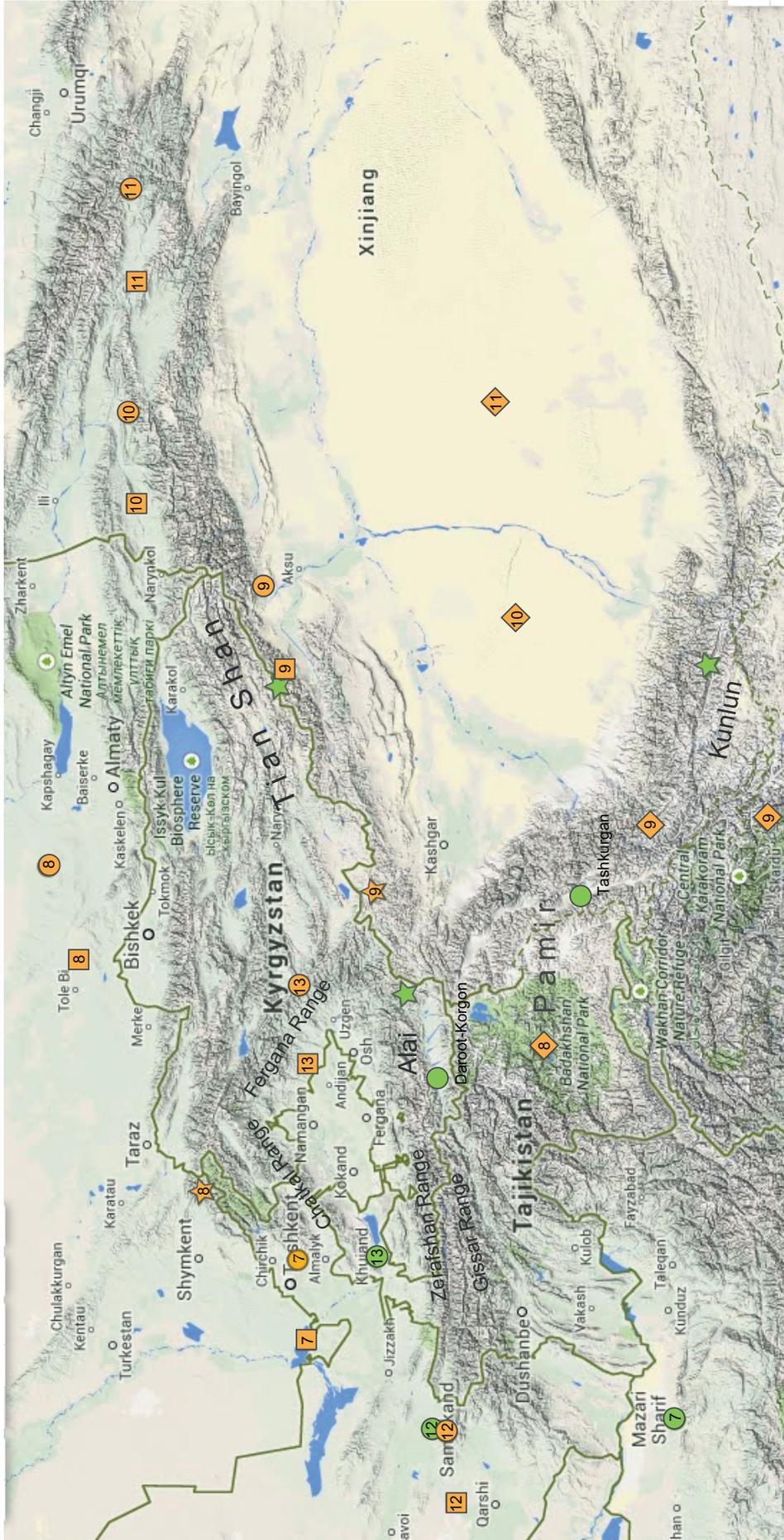


Fig. 4. Positions recalculated relative to Alexandria (yellow squares), to *Hierapolis/Membidj* (yellow circles) and to *Baktra* (yellow rhombi). Ascent to the range of *Komedai* and the gorge of *Komedai* are given also recalculated relative to *Alexandria Eschate* (yellow stars).

Map data ©2015 AutoNavi, Basarsoft, Google, Mapa/GISrael, ORION-ME, ZENRIN.

The Problems Outlined

The last station in the route to the Stone Tower mentioned by Ptolemy in discussing this *itinerarium*, *Baktra*, is placed by him not only onto a wrong latitude, but also into a wrong valley (see Fig. 4). In fact, the Ptolemaic map of Sogdiana shows only one of the important mountain ranges separating the land of the Sakai from Sogdiana, that is, the *Komedai* mountains (see Fig. 6).

Therefore, what is meant by Ptolemy for *Komedai* mountains, remains ambiguous. It may be interpreted as the Alay and Turkistan ranges which separate on the north and east the valley where *Baktra* is in fact situated and where (near Termez) an important crossing on the Amu Darya river was in use during ancient times. The consequence of such an interpretation would be both the complete absence of the Fergana valley – an enormous depression between the Tian Shan and Alay mountain systems – on Ptolemy's map and the wrong position of *Alexandria Eschate* between then unidentifiable mountain ranges.

But the *Komedai* mountains can also be interpreted as the mountain range which borders the Fergana valley, on the northwest by the Chatkal and Kuramin ranges, on the northeast by the Fergana mountains. The Fergana valley is framed by two rivers, the Naryn and the Kara Darya which join near Namangan, forming the Syr Darya river. The *Komedai* mountains on Ptolemy's map constitute in fact a region where (apart from the not yet identified *Baskatis* and *Dymos* rivers) also the *Iaxartes*/Syr Darya has its source. In favour of such an interpretation speak the positions of the ascent to the *Komedai* mountains (at the westernmost parts of the range) and of the gorge of the *Komedai* (at the southern and the eastern part) recalculated relative to *Alexandria* and *Hierapolis*/Membidj (see Fig. 4).

Although Ptolemy remarks that

... [Marinus] places the northern and the westernmost parts of the range, where the ascent is, on the parallel through Byzantion, and the southern and the eastern parts on the parallel through the Hellespont,

he places the gorge of the *Komedai* not at the *Hellespont's* latitude (about 41°) but at 39° in his catalogue without any explanation. The Ptolemaic revision of the positions of the intermediary stations along the route towards the *Sera Metropolis* is given schematically in Fig. 5.

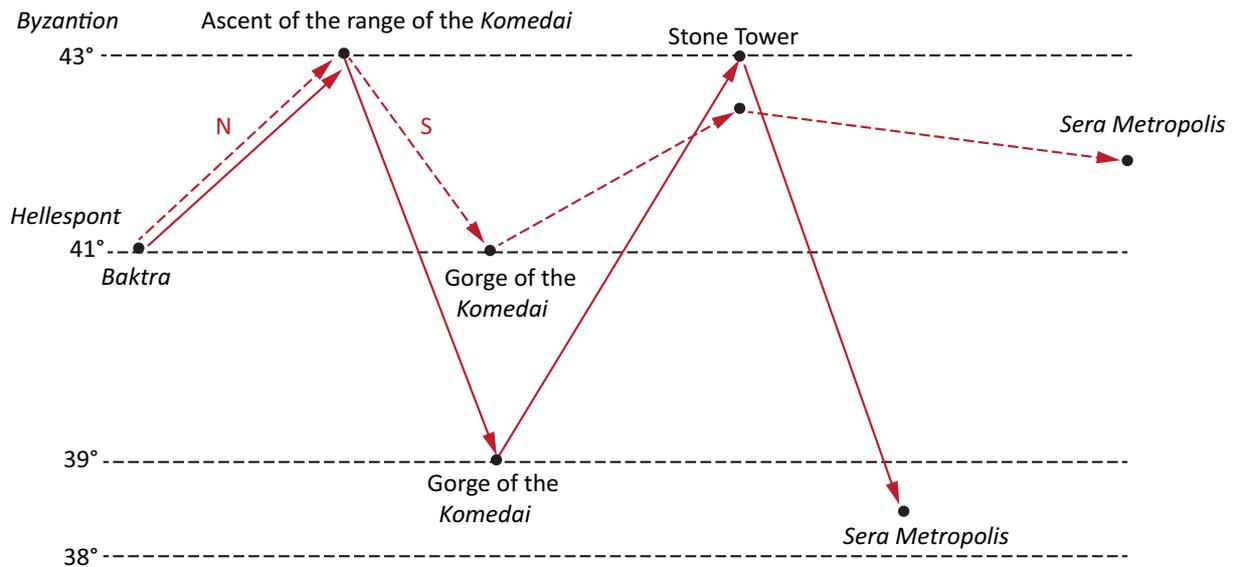


Fig. 5. Ptolemy's evaluation of Marinus' data (solid vs. dashed lines).

Trying to trace back the initial shape of the *Komedai* mountains in Marinus' work, we have recalculated these positions relative to *Alexandreia Eschate*. The results show that the main direction as well as the size of the mountains display the real position of the mountains which border the Fergana valley very well (see Fig. 4, the ascent of the range of the *Komedai* and the gorge of the *Komedai* are marked with yellow stars numbered 8 and 9, respectively).

The problem with such an identification becomes obvious, when we look at other details of the Ptolemaic map (Fig. 6). The rivers which originate in the *Komedai* mountains flow wrongly into the northern direction, joining in a region limited by the *Tapura* and *Sogdiana* mountains. If the *Tapura* mountains are identified with the Karatau mountains, there is no modern counterpart to the *Sogdiana* mountains. It is also hardly conceivable that Ptolemy placed such an important city as *Alexandreia Eschate*/Khujand, the most northern city founded by Alexander the Great in 327 BC, into the wrong valley far off the *Iaxartes*. What is more, it is then disconnected from the nearby *Kyreschata* (at Ura-Tjube to the southwest of Khujand) which lies on the Ptolemaic map to the north of it, close to *Iaxartes*.

It seems plausible that we deal here with a contamination of the Fergana and *Baktra* valleys with a possible double of the Fergana valley in the north of it, between the *Tapura* and *Sogdiana* mountain ranges. Regarding this possible confusion, the displacement of the adjusted positions would have occurred only too easily.

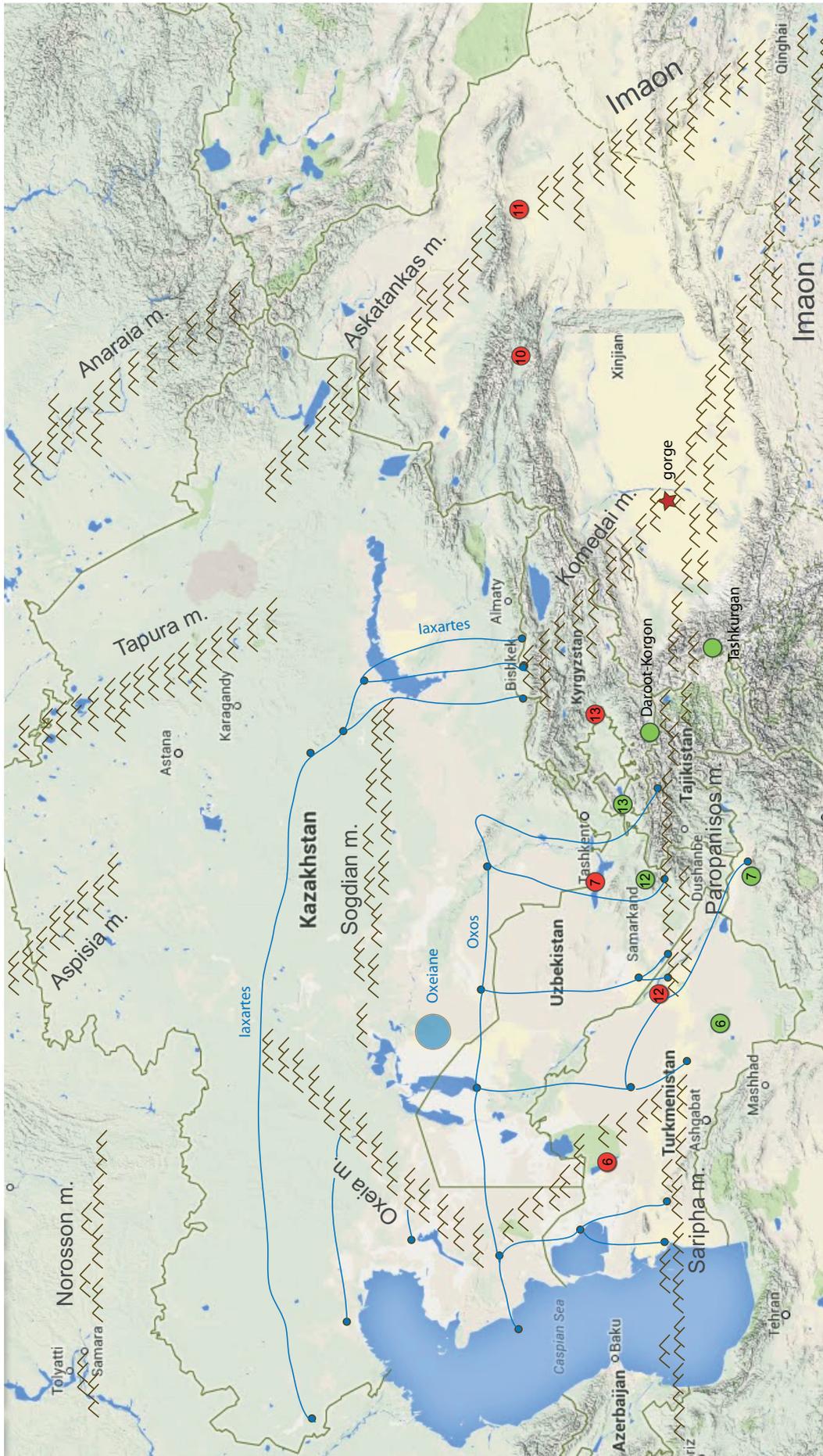


Fig. 6. Ptolemaic landscape of the region. The position of the Greenwich Meridian is defined by identifying the Ptolemaic longitude of *Baktra* with the longitude of Balch.

Map data ©2015 AutoNavi, Basarsoft, Google, Map/GISrael, ORION-ME, ZENRIN.

The interpretation of the journey towards the *Stone Tower* depends inevitably on the adopted identification of the *Komedai* mountains.

In the standard works of A. Herrmann,²⁰ the journey from *Baktra* to the *Stone Tower*, identified as Daroot-Korgon in the Alay Valley, was described according to the first interpretation of the *Komedai* mentioned above. The shortest and standard way from *Baktra* towards the Tarim basin goes in the northeastern direction towards the range of the Zarafschan mountains, then, after reaching the mountains, it deviates towards east and goes along the tale of Vakhsh river through the Trans-Alay Range until the Alay Valley. The Alay Valley is ca. 180km long in east-west direction and ca. 40km wide in north-south direction and lies at an altitude of 2500–3500m. In the eastern part of the valley there is the low Tongmurun pass leading towards the border crossing to China.

This journey digresses notably from the one described and discussed by Ptolemy. In order to come to grips with the problems at least partly, Herrmann proposed a huge digression of the route in the south of Gharm (approximately, where the Pamir Highway M41 can be found now). The proposed digression would go, among other things, through the Akbei-Sagran passage at an altitude of 4480 m which due to the information of *Zeitschrift des Deutschen und Österreichischen Alpenvereins* Bd. 45 (1914) is unsuitable for horsemen (the route to *Sera Metropolis* was surely used by caravans) followed by the gorge of Muksu which Herrmann identifies as the gorge of *Komedai*.²¹ After that, according to Herrmann, the route turns once more in the northern direction and enters the Alay Valley from the southern part followed by the standard route eastwards towards the Tarim Basin. In the footsteps of Richt-hofen, Herrmann identifies the *Stone Tower* with the modern Daroot-Korgon (see Fig. 4)

The even bigger problem consists in the distance to the *Stone Tower* extracted from Ptolemy's coordinates. It is overestimated at least by a factor of three. For the purpose of adjusting the known distances on his map, Ptolemy was not only forced to transfer the southern part of the *Komedai* mountains, the gorge of *Komedai*, two degrees to the south in contrast to Marinus, he also increased the size of the land of the Sakai to enormous proportion, measuring now ca. 16° x 13°.

Before further consideration, let us point out here that the all too common scholarly habit of avoiding a mathematical discussion of the Ptolemaic coordinates in this region seems always to be complemented by citing ancient references for the variety of relations between the lengths of the *stadion* and

²⁰ Herrmann 1910; 1968 (1938).

²¹ Herrmann 1968 (1938): 104.

the *schoinos*. It is true that these units vary from 1 *schoinos* being equivalent to 30 *stadia* to 1 *schoinos* equivalent to 120 *stadia*.²² But from all attested relations, the one chosen by Ptolemy – 1 *schoinos* equals 30 *stadia* – gives the *smallest possible distance* between the positions whose mutual distance was measured in *schoinoi*. All the other ratios for calculating longitudinal and longitudinal differences in angular values would produce an even *bigger* extension of the Ptolemaic maps. In other words: pointing to different values for the *schoinos* does only obfuscate the underlying problem and is of no help for the explanation of the peculiarity of the Ptolemaic positions at all. The problem of the far too big distances from *Baktra* to the *Stone Tower*, for instance, is clearly not one of a wrong conversion, i.e. that of distances expressed in *schoinoi* to distances in *stadia* followed by a calculation of the geographical coordinates.

Let us recall that, contrary to the usual practice of Ptolemy, the distances from the *Crossing of Euphrates* to the *Stone Tower* were only slightly modified: Ptolemy adopted the value of 876 *schoinoi* given by Marinus and reduced it to 800 *schoinoi*. We may attribute this to the fact that the Ptolemaic latitudinal position of such important stations along the route as *Alexandreia Margiane*/Merv and *Baktra*/Mazar-e Sharif are too high. By reducing the distances transmitted by Marinus by a third – which seems to have been his standard practice – Ptolemy would not have been able to adjust the positions lying at these overestimated latitudes. By adopting Marinus' values without much reduction along the route from *Hieropolis* to *Baktra*, Ptolemy was consequently forced to keep also the length of the route from *Baktra* to the *Stone Tower*.

After that, Ptolemy (possibly) following Marinus misinterpreted the descriptions of the routes towards the *hormeterion*.

Three possible alternative scenarios can be formulated.

A) The locations and distances gleaned from Ptolemy's text belong to a northern route of the Silk Road which goes to the north of *Baktra* up to the northern ranges of the Tian Shan mountains. Therefore, the ascent to the *Komedai* mountains now lies approximately at its position recalculated relative to *Alexandreia Eschate* (see Fig. 4). There are plenty of ancient routes between Shymkent and Bishkek which allow an entrance to the high plain of Kyrgystan. This region must then be equivalent to the land of the Sakai. Through the Bedel passage an old route, used also by Chinese, allows to enter the Tarim basin in the vicinity of Ukturpan and Aksu. The other possibility would be the passage in the north of Kashgar (now the standard route between Kyrgystan and China). In this case Kashgar (Kashi) is confirmed as the standard interpretation of the *hormeterion*. Such a proposal cancels out the

²² Hultsch 1971 (1892).

interpretation of Daroot-Korgon as the *Stone Tower*. One has to look for a different identification.

B) The agents of Maes Titianos did in fact take the route to Alay Valley, which was widely used in Antiquity, through the Fergana valley up to Osh, followed by a picturesque gorge to the *Stone Tower*/Daroot-Korgon. *hormeterion* would lie in the vicinity of (if not all at) Kashgar. Because both the Zerafshan Range and the Gissar Range were absent on his mental map of the region, Ptolemy (Marinus) adjusted the description to the *Komedai* mountains.

The journey from *Baktra* in the northern direction to the ascent to the range of the Zarafshan mountains went possibly through some slopes (if not around) to Fergana Valley which is the “plane” in Ptolemy’s text and from Uch-Korgon in the southern direction over the Tengizbei pass and to the Tengizbei gorge connecting the Fergana Valley with the Alay Valley. An alternative branch of the route which leads down to the Alay Valley goes through Uzgen over the Terek Pass toward a major fork in the route near Irkeshtam. The misattribution might have happened already in the work of Marinus: the information about the ascent to the range of the mountains, followed by a plane with a gorge leading to the *Stone Tower* may have been adjusted on his map to the only mountain range available – and, unfortunately, this was the wrong one. In favour of such an interpretation speaks the very good match of the position of *hormeterion* with Kashgar, if we recalculate its coordinates relative to the *Stone Tower* identified as Daroot-Korgon (see Fig. 7).

C) After leaving *Baktra*, Maes’ agents used a way through the high plateau of the Pamir following the very well known merchant route towards India. The positions of the ascent to the range of the *Komedai* and the gorge of the *Komedai* recalculated in relation to *Baktra* are given in Fig. 4 and marked with yellow rhombi numbered 8 and 9, respectively (the upper rhombus represents the position of the gorge according to Marinus). In this case, the *Stone Tower* can be identified with one of the earlier proposals in modern studies – Tashkurgan (see Fig. 4)²³ and the expedition entered the Tarim basin from the south-eastern part between Hotan and Yarkant. The recalculated position of *hormeterion* in this scenario should lie near the Pishan Oase (see Fig. 7). This place was also known in the annals of the western Han Dynasty (see item 5 in Table 1 of the next section). The direction of the route described by Marinus is in favour of this scenario.

In all scenarios, due to the misinterpretation of the mountain range, the route to the *Stone Tower* is not adjusted properly to *Baktra* and to locations along the route mentioned by Ptolemy. Thus, a recalculation of coordinates in *Skythia on this side of the Imaon* cannot be performed relative to these loca-

²³ This is also the identification for the *Stone Tower* in Stückelberger/Grasshoff 2006.

tions. In fact, due to the overestimated distances, even after our recalculation to the Eratosthenian size of the Earth relative to *Baktra* as a reference point, the *Stone Tower* and the *hormeterion* lie too far to the east from their presumed locations in the Tarim basin. As a consequence, the Ptolemaic distances in this region should be considered and interpreted separately. In the best case, they can be adjusted to the utmost western location used in the mapping procedure, that is to the *Stone Tower*.

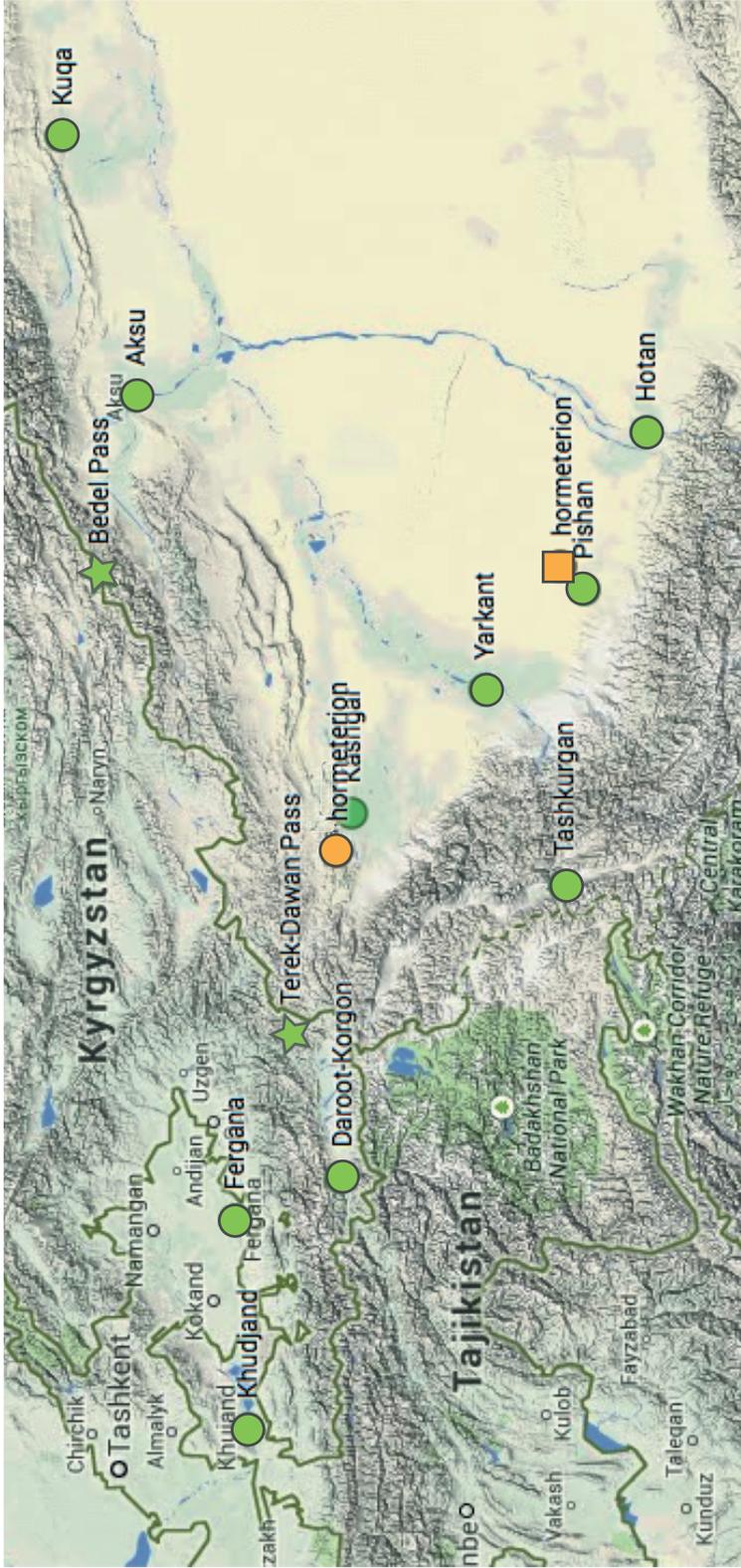


Fig. 7. Positions of the *hormeterion* recalculated relative to the *Stone Tower*/Daroot-Korgon (yellow circle) and to the *Stone Tower*/Tashkurgan (yellow square).

Map data ©2015 AutoNavi, Google.

The Tarim Basin in the Annals of the Han Dynasty

The identification of the locations in Ptolemy's *Scythia on this side of the Imaon* and *Serike* poses a special problem, since the Ptolemaic data alone are not sufficient to provide a single and undisputable solution. The necessity to take into consideration also the Chinese sources for the region in question was therefore already understood in early studies devoted to this topic.²⁴

Two Chinese primary sources include invaluable information about the Western regions and the background of the network of available connecting routes. The *Shiji* is devoted to a general history of China; it starts from early Antiquity and covers the time until ca. 90 BC when it was probably completed.²⁵ The principal compiler of the work was the famous historian Sima Qian. Chapter 123 of the *Shiji*, entitled "The Memoir on Dayuan", includes information collected by Zhang Qian who built new relations to the states of the Far West during his journeys; the chapter describes, besides the land in case, also many other adjoining states. The places mentioned in the text were not linked with the Northern or Southern routes of the Silk Road which had not yet evolved in Zhang Qian's time.²⁶

The second source are the annals of the Former, or Western, Han Dynasty, *Han shu*, which cover the period from its foundation in 202 BC to the fall of Wang Mang in 23 CE.²⁷ The compilation of the *Han shu* was started about 36 CE and probably completed by the historian Pan Ku between 110 and 121 CE.²⁸ The entire 96th chapter of these annals is devoted to a description of the Western Regions.²⁹ The main part concerns the geographical position and size of the countries, but also provides some information about the landscape features, number of citizens, the course of the main streets as well as important distance data. Some distances are given relative to the contemporary Chinese capital Chang'an, other relative to the residence of the Protector General³⁰ of the Western Regions in Wulei. In addition, for some countries, the distances of their capitals to their neighbouring countries are also listed. These distances are measured along the connecting routes; some of them are

²⁴ See, e.g., D'Anville 1768: 572–603; Ritter 1837: 559; Cunningham 1847: 989; Lassen 1874: 540; Richthofen 1877: 479; Herrmann 1910.

²⁵ Hulsewe 1979: 8.

²⁶ For a discussion see, e.g., Hill 2004.

²⁷ Hulsewe 1979: 197, note 711.

²⁸ Hulsewe 1979: 8.

²⁹ For the period for which the two histories overlap, however, they do not present different or independent information. For the peculiar relation between the two monographs see, e.g. Hulsewe 1979: 11ff.

³⁰ For the genesis of the position of the Protector General see Loewe 1974: 228–230.

complimented with one of the eight cardinal directions. The standardized sequence of data is:³¹

1. The name of the country;
2. The name of the seat of the ruler's government;
3. The distance of the latter from Chang'an;
4. The remark that the country is not subordinate to the Protector General;
5. The number of households, inhabitants and men capable of bearing arms;
6. The distance to the seat of Protector General, including the direction;
7. Similar data regarding neighbouring countries;
8. The countries with which the state under discussion has common borders;
9. A description of its customs and its products.

It is therefore clear that this manner of discussion corresponds to the approach which Ptolemy (*Geogr.* 1.1) called *chorography*, *χωρογραφία*. In which way this information was gained becomes clear from the text itself:³²

Since the time of Emperors Hsüan [=Xuan] and Yüan, [=Yuan] the *Shan-yü* [=Shanyu] has styled himself vassal and the Western Regions have been submissive. [So] the extent of the lands and the Western Regions, their mountains and rivers, their kings and nobles, the numbers of their households and the distances by road have become clearly known.

What was understood under the Western Regions can be glimpsed from the very beginning of chapter 96 of the *Han shu*:

Communications with the Western Regions started only in the time of Emperor Hsiao Wu. Originally there had been only thirty-six states, but afterwards these were gradually divided into more than fifty. These all lie to the west of the Hsiung-nu [=Xiongnu] and south of Wu-sun [=Wusun]. To the north and south there are great mountains, and a river flows through the middle. The distance from east to west extends for more than 6000 *li* and from north to south for more than 1000 *li*.

On the east the area adjoins Han [territory], being blocked by the Yü-men [=Yumen] and the Yang barriers. On the west it is confined by the Ts'ung-ling [=Congling]. Its southern mountains emerge in the east in Chin-ch'eng [=Jincheng] [commandery] and are linked with the Nanshan of Han. Its river has two sources, of which one rises in the Ts'ung-ling and the other in Yü-t'ien [=Yutian]. Yü-t'ien lies at the foot of the southern mountains, and its river runs northward to join the river that

³¹ Hulsewe 1979: 124, n. 299.

³² *Hanshu*, chapter 96A, 3B; the translation is given according to Hulsewe 1979.

comes from the Ts'ung-ling. Eastward it flows into the P'u-ch'ang [=Puchang] Sea. Another name of the P'u-ch'ang Sea is the Salt Marsh; it is [1]300 *li* distant from the Yü-men and Yang barriers and measures 300 [to 400] *li* in width and length. Its water remains stagnant, and are not increased or reduced in winter or summer. It is said: "It is generally believed that the water flows hidden below ground, and that it emerges to the south at Chi-shih [=Jishi] to form Ho [He] of China."³³

The geographical names mentioned in the account are the following: Congling – the Pamir mountains, Nanshan – the mountains south of modern Xi'an (former Chang'an), Yutian is Hotan, Puchang Sea is Lob-nor, He is the Yellow River, the Yumen and the Yang are the barriers in the Great Wall at the western terminal of the Han defence.³⁴ The primary identification of the West Regions with the territory of the Tarim basin is therefore straightforward.

The compiler proceeds with discussion of the routes leading through the Western Regions:

Starting from the Yü-men and Yang barriers there are two routes which lead into the western Regions. The one which goes by way of Shan-shan [=Schanshan], skirting the northern edge of the southern mountains and proceeding along the course of the river west of So-chü [=Suoju] (Yarkant) is the southern route. To the west, the Southern Route crosses the Ts'ung-ling (Pamir) and then leads to Ta Yüeh-chih [=Da Yuezhi] (Baktria) and An-hsi [=Anxi] (Parthia of the Arsacids). The one which starts from the royal court of Nearer Chü-chih [=Juzhi] (Turfan), running alongside the northern mountains and following the course of the river west to Shu-lo [=Shule], (Kaschgar) is the Northern Route. To the west, the Northern Route crosses the Ts'ung-ling and leads to Ta Yüan [=Da Yuan] (Ferghana or Sogdiana?), K'ang-chü [=Kangju] (Samar-kand?) and Yen-ts'ai [=Yancai].

Thus, with great probability the Northern Route was assumed to head over the Terek-Dawan or the Tengizbei Pass to Fergana. It is not clear, whether both routes joined together in the Alai Valley or, if not, the Southern route should proceed towards *Baktra* through Tashkurgan and the passes of the Pamir Mountains.³⁵

The distances incorporated in the text are of primary importance for identifying the old Chinese locations. A short account is given in Table 1.

³³ *Hanshu* chapter 96A; p. 71ff.

³⁴ For a discussion regarding the location of the barriers see, e.g., Hulsewe 1957: 1–50.

³⁵ This interpretation is supported by our results discussed in the section "Interpretation of the Ptolemaic positions in the Tarim Basin" below.

N. Location seat of the king's government identification	Distances (in <i>li</i>)		Population
	to Ch'ang-an [=Chang'an]	to Wu-lei [=Wulei]	

1. Wu-lei [=Wulei]	9950	-----	1200
2. Ch'ieh-mo [=Qiemo] near Cherchen	6820	2258 NW	1610
3. Ching-chüeh [=Jingjue] Cadota in Niya site	8820	2723 N	3360
4. Yü-T'ien [=Yutian] West City Chotan	9670	3947 NE	19300
5. P'i-shan [=Pishan] P'i-shan	10050	4292 NE	3500
6. Hsi-yeh [=Xiye] valley of Hu-chien [=Hujian] Yularik, south of Yarkant	10250	5046 NE	4000
7. So-chü [=Suoju] Yarkant	9950	4746 NE	16373
8. P'u-li [=Puli], I-nai [=Yinai] valley of P'u-li [=Puli] Tashkurgan?	9550	5396 NE	5000
9. Shan-shan [=Schanschan] Wu-ni [=Wuni] Charkhlik?	6100		14100
10. Hsiao-Yüan [=Xiaoyuan] Wu-ling [=Wuling] south to Charkhlik	7210	2558 NW	1050
11. Wu-mi [=Wumi] / Chü-mi [=Jumi] Wu-mi [=Wumi] Kara-dang?	9820	3553 NE	20040
12. I-nai [=Yinai] near Yarkant	10150	2730 NE	670

13. Nan-tou [=Nandou] lower course of Gilgit	10150	2850 NE	31000
14. Wu-i-shan-li [=Wuyishanli] Alexandria?	12200	60 day NE	
15. An-hsi [=Anxi] Fan-tou [=Fandou] the Arsacides, i.e. Persia	11600		Several hundreds towns
16. Ta Yüeh-chih [Da Yuezhi] Chien-shin [=Jianxin]	11600	4740 E	400000
17. K'ang-chü [=Kangju] Samarkand's region	12300	5550 E	600000
18. Ta Yüan [=Da Yuan] Kuei-shan [Guishan] Fergana's region	12250	4031 E	300000
19. Hsiu-hsün [=Xiuxun] Niao-fei [=Niaofei] valley Alai valley?	10210	3121 E	1030
20. Chüan-tu [=Juandu] Yen-tu [=Yandu] Near Irkeshtam?	9860	2861 E	1100
22. Shu-lo [=Shule] Shu-lo [=Shule] Kashgar	9350	2210 E	18647
23. Wu-sun [=Wusun] Ch'ih-ku [=Chigu] Ili valley? South of Tekes?	8900	1721 E	630000
24. Ku-mo [=Gumo] Nan Aksu areal. Uch-Turfan?	8150	1021 E	24500
25. Ch'iu-tzu [=Qiuzu] Yen [=Yan] Kuqa	7480		81317
26. Ch'ieh-mo [=Qiemo] Ch'ieh-mo [=Qiemo] Cherchen area	6820	2258	1610
27. Ch'ü-li [=Quli] SE of Kucha. Charchi?			1480

28. Wei-li [=Weili] Wei-li [=Weili] NE of Korla? Bugur?	6750	300 W	2000
29. Mo-shan [=Moshan] Mo-shan [=Moshan] North of Lop Nor	7170	240 W	5000
30. Nearer Chü-shin [=Juxin] Chiao-ho [=Jiaohe] Turfan area	815	1807 SW	6050
31. Further Chü-shin [=Juxin] Jimsa?	8950	1237 SW	4774
32. Wen-su [=Wensu] Wen-su [=Wensu] Uch in Aksu	8350		8400
33. Yen-ch'i [=Yanzhi] Yüan-ch'ü [=Yuanqu] Karashar	7300	400 SW	32100
34. Wei-li [=Weili] Wei-li [=Weili] Bugur, Korla?	6750	300 W	9600

Table1. Distance Data in *Han shu*.

Fig. 8 shows the position of identified locations discussed in *Han shu*. The courses of the routes connecting the Han Dynasty with the western countries are noticeably marked by locations situated along these routes.

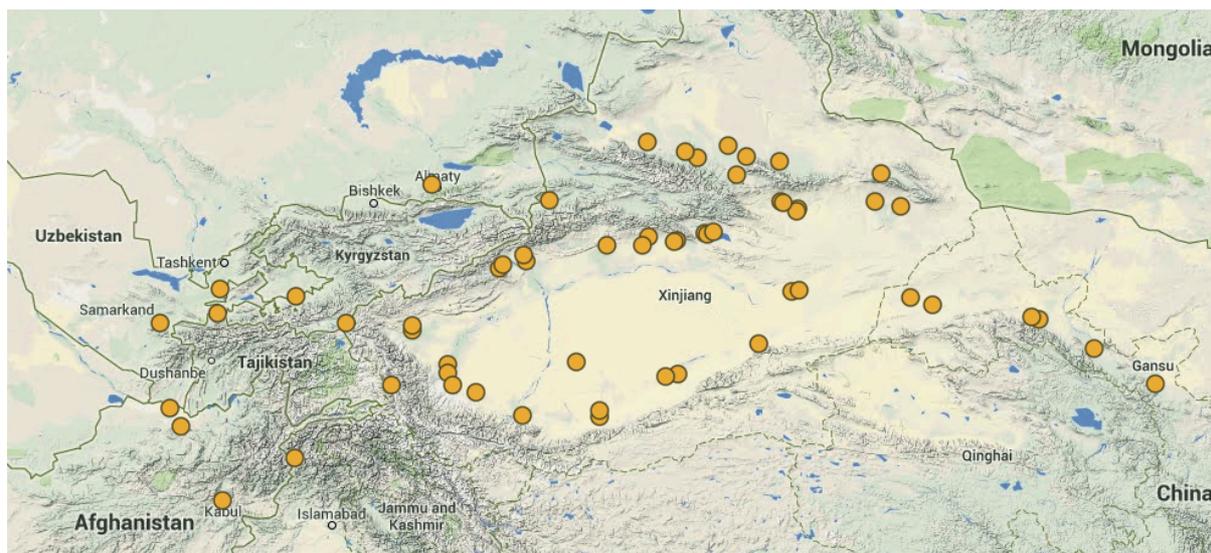


Fig. 8. Identified locations mentioned in *Han shu*.

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Two Han Histories address the main routes as the “Northern route” and the “Southern route”. In fact, there were plenty of different branches of the connecting routes and the choice between them was often made on the basis of the political situation or weather conditions. For example, a short but difficult route between Dunhuang and Shanshan led directly across the desert; it was substituted by a longer route through the high Altin-tagh range during the part of the year when the desert route could not be used because of the heat.³⁶ Another example is the branch of the southern route which started at Cherchen and went south through the mountains, across the Qaidam marches, around the southern shore of Koko Nor via Xining to Lanzhou and farther to central China. An ancient turn-off towards Lhasa in the western part of Koko Nor at the junction of the trails was more frequently used. The state of Xiao Yuan was, according to *Han shu*, a three days’ march south of modern Cherchen. The “capital” of this state, Yuling, might be placed near Dalai-kurghan (as Herrmann proposed) or near modern Tura or Bash Mulghun “which control a valley of rich grasslands, easily-defended and guarding the junction of two important routes”.³⁷ The traces of these routes may possibly be detected in the Ptolemaic coordinates of the regions.

Additional information about the ancient connecting routes in use can also be gained from the text of the *Weilue* or “Brief account of the Wei Dynas-

³⁶ Stein 1912: 514–515.

³⁷ Hill 2004: Sec. 4, n. 5.3.

ty” by Yu Huan. Although the original text has been lost, the chapter on the “People of the West” was quoted in detail by Pei Songzhi, first published in 429 CE. An English translation with good comments is available in Hill 2004. As the translator states, “the *Weilue* contains much new, unique, and generally trustworthy material. Most of it dates appears to have come from the Later Han dynasty, before China was to a large extent cut off from the West by civil wars and unrest along its borders during the late 2nd century CE.” The description implies that what was called the “Northern route” in the two Han Histories, is called the “Central route” in the *Weilue* and for the “Northern route” of Han annals two branches, a “New route” and a “New route of the North” are discussed. *Weilue* reports also the existence of a route to the north of Taklamakan Desert (which is obviously discernible in Fig. 8, but is not described as such in the *Han shu*); this route was likely preferred during times of rather stable connections with the Xiongnu.

The “Central route” or “Middle route” of the *Weilue* started at the “Jade Gates” (Yumen), headed northwest to Loulan (at Lop Nor) and then proceeded along Kuruk Darya and Konche Darya (now dry) and joined at modern Korla the branch of the route coming from Turfan. It proceeded further to Kuqa and Aksu where it split in two branches: a route in direction of Kashgar and a route which led over the Bedel Pass to the old Wusun territory and to the Issyk Kol regions. There were also north-south connections across the Tarim basin such as one linking Hotan and Aksu.

This network of roads was called the “Silk Road(s)” by F. von Richthofen in 1877. A map based on information collected since more than a century by the Berlin Academy project “Turfan Studies” (coordinated by Desmond Durkin-Meisterernst) illuminates the linguistic plurality in the region³⁸ It also shows how the Silk Road was not only a trading route, but also a means of cultural and religious exchange.

Another characteristic of locations – the size of population – can tell much about the significance of a city; it may also correspond to the Ptolemaic classification of “important” locations.

³⁸ *Turfan Studies* 2007: 20–21.

Interpretation of the Ptolemaic Positions in the Tarim Basin

Let us start with the following undisputable statement: Ptolemy put a position on his map only relative to another one, which we call reference point. It is therefore obvious that the first step in reconstructing the Ptolemaic map is the choice of the correct reference point(s) used in his mapping procedure. *Appendix 1* and *2* of the present paper consider the problem from the mathematical point of view: one result, in short, is that the mathematical consequences of mapping on a spherical Earth of wrong size and those of an erroneous recalculation of insufficiently known measuring units (such as *li*, *parasangs*, *schoinoi*, etc.) are the same. Cases, where the same locations pop up under different names, when positions change their mutual orientation or when locations seem to be erroneously linked with nearby positions can in many instances be interpreted as a simple cartographical manifestation of the erroneous presuppositions mentioned above in conjunction with the use of more than one reference point. In *Appendix 2* these kinds of errors are called “doubling”, “inversion” and “overmapping”. In the region under discussion, whereof Ptolemy apparently had no information in regard to the latitudes, our algorithm boils down to transferring the angular distances and directions of positions relative to a selected reference point from a sphere with the circumference of 180,000 units to a sphere with the circumference of 252,000 units (Case 2 in Tupikova/Geus 2013).

The comparison of recalculated coordinates with actual positions of historical locations in this region will provide strong evidence for the appropriateness of our approach.

In the following maps we will give the positions mentioned for the map Asia 8 (*Geogr.* 6.15.1–6.16.8) following Ptolemy according to the following numbering:

1. Λίθινος Πύργος / *Stone Tower*,
2. ὀρμητέριον / *hormeterion*,
3. Αὐξάκια ὄρη / *Auzakia*,
4. Ἴσσηδὼν Σκυθική / *Scythian Issedon*,
5. Χαύρανα / *Chaurana*,
6. Σοῖτα / *Soita*,
7. Δάμνα / *Damna*,
8. Πιάδα / *Piada*,
9. Ἀσμιραία / *Asmiraia*,
10. Θροάνα / *Throana*,
11. Ἴσσηδὼν Σηρική / *Issedon of Serike*,
12. Ἀσπακάρα / *Aspakara*,
13. Δρωσάχη / *Drosache*,

14. Παλιάνα / *Paliana*,
15. Θογάρα / *Thogara*,
16. Άβραγάνα / *Abragana*,
17. Δαξάτα / *Daxata*,
18. Όροσάνα / *Orosana*,
19. Όττοροκόρα / *Ottorokora*,
20. Σολάνα / *Solana*,
21. Σήρα μητρόπολις / *Sera Metropolis*.

The following cities are marked by Ptolemy in *Scythia on this side of the Imaon* and *Serike* as important: *Auzakia*, the *Scythian Issedon*, the *Issedon of Serike*, *Drosache* and *Sera Metropolis*.

As we have already shown, the westernmost reference point which can be properly linked with the Ptolemaic positions in this region is the *Stone Tower*. We will consider first the interpretation of the *Stone Tower* as Daroot-Korgon.³⁹ In this case, the position of *hormeterion* is a place in the vicinity of Kashgar (see Fig. 7).⁴⁰ Since we do not know which point exactly was considered by Ptolemy as the “starting point for trade with Serike”,⁴¹ but need a real and accurately chosen geographical location of a reference point for our recalculations, we have selected Daroot-Korgon for the first attempt to visualise the Ptolemaic positions recalculated to Eratosthenes’ size of the Earth. The position of the Greenwich Meridian is then defined through identification of the longitude of the *Stone Tower* with the longitude of Daroot-Korgon. The original Ptolemaic positions referred to Daroot-Korgon are given in Fig. 9.

At first sight it is not obvious at all, why the positions for this region should be identified with the locations in the Tarim basin: whereas coordinates in the southern part of the map approximately match the latitudes of the locations lying in south of the Tarim basin, the northern positions of Ptolemy lie partly in Mongolia and up to the latitude of the Baikal Sea. In fact, before the pioneering studies of J. B. D’Anville, the positions in the Asia 8 map were often relegated to Tibet, Mongolia or Western Siberia.⁴² Only for *Sera Metropolis* were the proposals (relatively) uniform, since it has been identified by the majority of modern scholars with the old Chinese capital Chang’an (modern Xi’an) or with one of the cities in the Gansu corridor leading to

³⁹ As proposed by A. Stein in Stein 1928: 848–850, 893–894.

⁴⁰ Stein 1928: 850 makes a strong case for identifying *hormeterion* as modern Irkeshtam, about 200 km west of Kashgar.

⁴¹ As ὀρημέτριον was, surely correctly, phrased in Stückelberger/Grasshoff 2006.

⁴² Ferguson/Keynes 1978: 584 still identifies Ptolemy’s *Oichardes* with Yenisej and *Bautisos* with Huang He (the Yellow River).

Chang'an, that is, with Ganzhou⁴³ or Liangzhou (Wuwei).

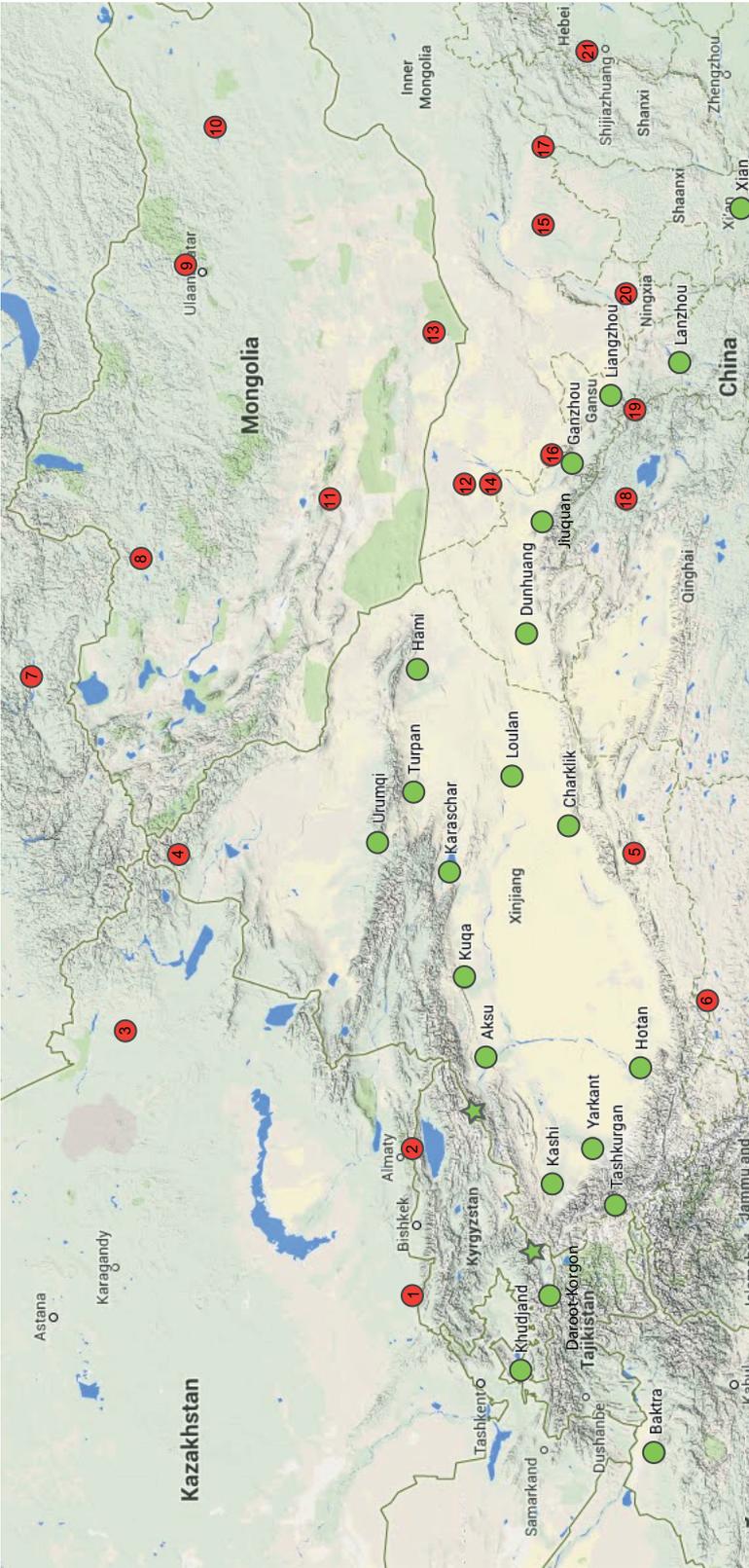


Fig. 9. Ptolemaic positions linked to the position of the Greenwich Meridian defined through identification of the *Stone Tower* with Daroot-Korgon.

Map data ©2015 AutoNavi, Basarsoft, Google, Map/GISrael, ORION-ME, ZENRIN.

⁴³ The city Ganzhou is now known as Zhangye. Marco Polo, who spent a year in this city called it *Campichu*.

According to A. Herrmann, this “overextension” originated in the doubling of the travel time to *Sera* of seven months (which, according to him, was meant as a round-trip time), followed by an estimation of the whole route to *Sera* as 36,200 *stadia*.⁴⁴ He postulated also (in fact, never confirmed) the existence of an ancient “travel guide” with distances expressed in *li* and proposes that for some legs of the route there was no travel guide available and that Maes’ agents estimated these distances in their usual *parasangs*. In order to save the estimated distance to *Sera*, Marinus decided to recalculate the distances expressed in alien units (*li*) with the factor of 3 (instead of the correct factor 2). Herrmann’s artificial reconstruction, followed by erroneous recalculations of “Ptolemaic” distances,⁴⁵ has, nevertheless, gained much recognition by modern scholars and is considered as a solid, mathematically verified basis for farther identification proposals.

The nowadays widely adopted identifications of the Ptolemaic positions by Herrmann are given in Fig. 10.

The non-uniform pattern of these identifications is clearly visible.⁴⁶ To explain the far too northern latitudes for a part of the Ptolemaic positions, A. Herrmann asserted that the only available sources for this part of the Earth were adopted by Ptolemy directly from Marinus’ work and that the distortions of Ptolemaic map are due to his attempt to compress Marinus’ original map to a “preconceived value” of longitudinal extension of the *oikoumene*, 180°.

⁴⁴ Herrmann 1968 (1938): 109.

⁴⁵ Herrmann gives no hints as to the method how he recalculated the „Ptolemaic“ distances (in *stadia*) from Ptolemy’s own catalogue. According to our verification check, he used plane triangles and recalculated the routes by the Pythagoras’ theorem. After having obtained the lengths of routes in angular value in this way, he converted them into *stadia* by equating at different places either 500 *stadia* per degree or 400 *stadia* per degree (parallel of Rhodes). But a fatal mistake crept into his calculation, when he also recalculated the routes going approximately along a meridian. On Ptolemy’s Earth one degree measured along a meridian is always equivalent to 500 *stadia*!

⁴⁶ Also the inversion of the positions of *Daxata* (17) which Herrmann places at the *Yang Kuan* gate in the Great Wall near Dunhuang and *Drosache* (13) which he identifies as Jiuquan (Herrmann 1968: 138) breaks the pattern of identifications. Misled by this inversion, Lindegger 1993: map 3, in his otherwise highly informative works, even switches the two positions on a map representing the original Ptolemaic coordinates.

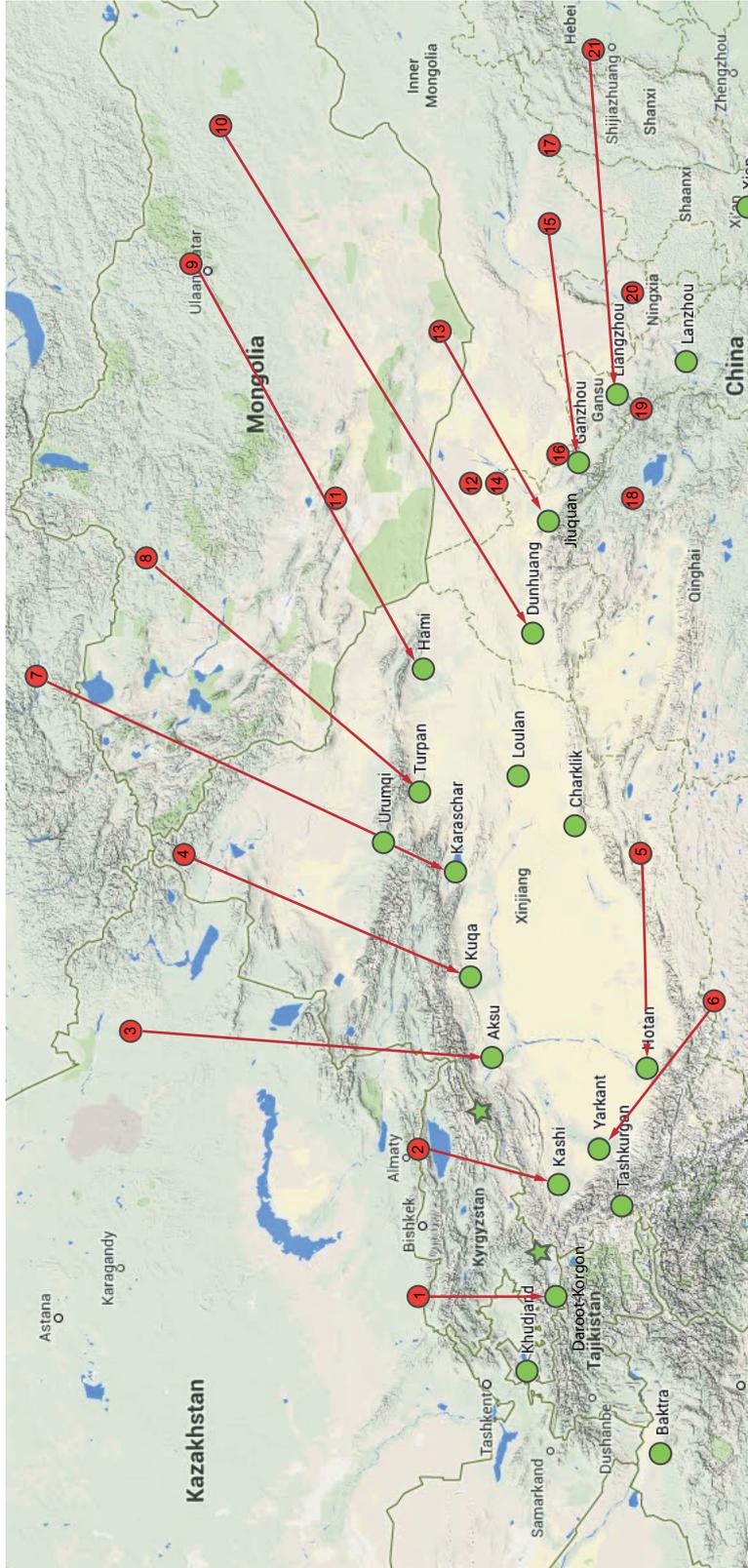


Fig. 10. Identification of the Ptolemaic positions according to A. Herrmann.

Map data ©2015 AutoNavi, Basarsoft, Google, Map/ GISrael, ORION-ME, ZENRIN.

Fig. 11 gives an impression of Ptolemy's landscape concept for the region under discussion.

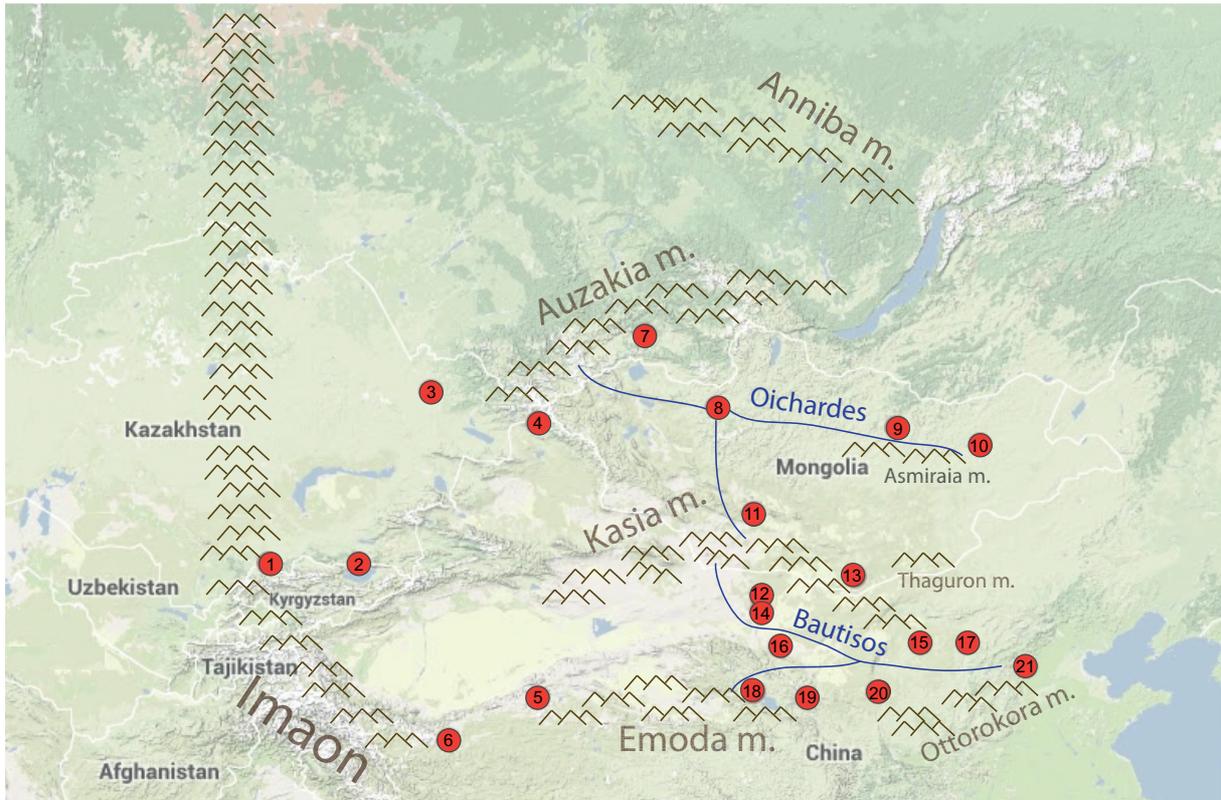


Fig. 11. Ptolemy's landscape and water system of the *Oichardes* and *Bautisos* rivers. The position of the Greenwich Meridian is defined relative to the *Stone Tower* / *Daroot-Korgon*.

Map data ©2015 AutoNavi, Basarsoft, Google, Mapa/GISrael, SK planet, ORION-ME, ZENRIN.

Of course, Ptolemy used to give only the utmost western and eastern positions for the mountain systems without information about their width; hence, one cannot be certain, e.g., whether Ptolemaic *Damna* lies in the *Auzakia* mountains or more to the south in a valley. The same problem holds true for the courses of the rivers: *Paliana* and *Orosana*, e.g., can lie on the other side of the river.

Although the position of the *Bautisos* river matches the (at least latitudinal) position of the Tarim river better than the one of *Oichardes*, it is the latter which is traditionally identified with the Tarim. The identification of the rivers *Oichardes* and Tarim can be traced back to the year 569 AD when a Byzantine delegation arrived at the river (*ard*) Tarim, which was called by the

natives *Oich*.⁴⁷

As a surprise comes the lack of information about the biggest influents of the *Oichardes*/Tarim River: in context of the system of contemporary identification of the Ptolemaic places, Kashgar, Aksu, Yarkant as well as Hotan rivers are not displayed on Ptolemy's map.

Fig. 12 displays the actual river system in the Tarim Basin.



Fig. 12. The water system in the Tarim basin (Wikipedia.org)

Whereas modern literature agrees about the identification of the *Oichardes*, the *Bautisos* river is not yet identified and is considered to be the Yellow River Huang Ho⁴⁸, Gez-Darya,⁴⁹ a reminiscence of the Brahmaputra,⁵⁰ or even a figment of Ptolemy's imagination.⁵¹

We will now try to verify whether the Ptolemaic coordinates recalculated to Eratosthenes' size of the Earth are able to help us with the identification of its positions. The results of our recalculation relative to the *Stone Tower*/Daroot-Korgon are given in Fig. 13.

The first striking result is that the position of *Sera Metropolis* now lies between Ganzhou (Zhangye) and Liangzhou, both well-known proposals for its identification.

⁴⁷ See Menand. Exc. de legat. p. 452.29 de Boor.

⁴⁸ Lindegger 1993: 83.

⁴⁹ Haussig 1959: 158ff.

⁵⁰ *Ptolemaios. Handbuch der Geographie*, eds. Stückelberger/Graßhoff 2006: 667.

⁵¹ Herrmann 1968 (1938): 122–3, Tafel 9.

Another important feature of the Ptolemaic map should be stressed here: one of the rarely disputed identifications of the Ptolemaic *Throana* as modern Dunhuang, inevitably lies on the route towards the cities which have been proposed as possible identifications for *Sera Metropolis* – these are the cities in the Gansu Corridor (Ganzhou, Liangzhou) or, farther to the south, Lanzhou and Xi'an. This route proceeds along the south-eastern direction. However, on the Ptolemaic map *Throana* lies directly to the north of *Sera Metropolis*. How is this apparent lack of connection between two locations lying along the same route to be explained?

At least at this stage one must put forward the inevitable question: did Ptolemy, when constructing his maps, possess and use information which was independent of Marinus' sources? If not, which reasons could have existed to convince Ptolemy to suggest, in contrast to Marinus, for instance that "the *Stone Tower* is near the parallel through *Byzantion*, and *Sera* is south of the parallel through the *Hellespont*" (*Geogr.* 1.12)?

What is also remarkable here, is that all the other positions in this scenario seem not to be linked directly with the *Stone Tower*/Daroot-Korgon. More than that, especially the nearby locations which are unavoidably bound to follow Kashgar on the northern route (*Auzakia* normally identified as Aksu) or on the southern route (*Soita* identified as Jarkant) show most prominent deviations from their proper positions!

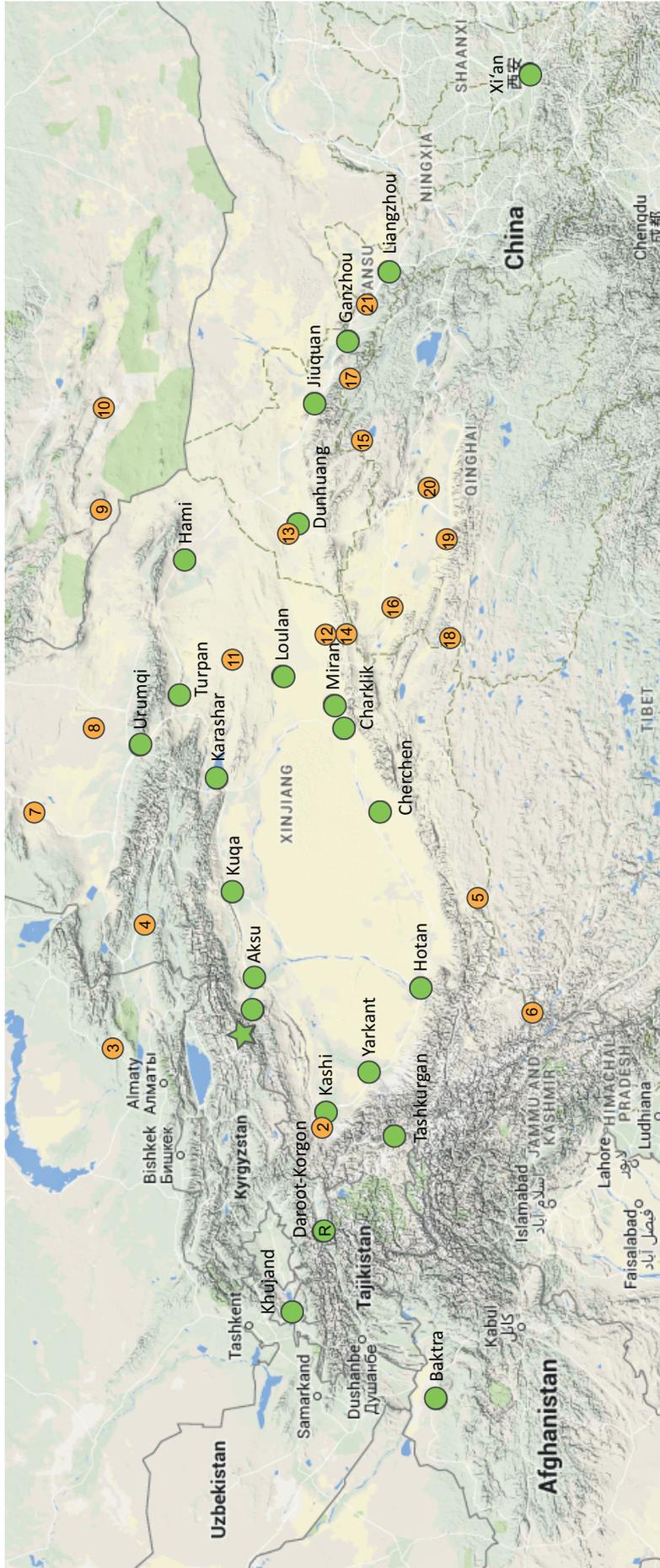


Fig. 13. Positions recalculated relative to the *Stone Tower* = Daroot-Korgon (marked by a circle with letter "R").

Map data ©2015 AutoNavi, Basarsoft, Google, Mapa/GISrael, ORION-ME, ZENRIN.

If the *Stone Tower* can be identified with Tashkurgan (37° 46' 22" N, 75° 13' 28" E) in Xinjiang, Maes' agents took the ancient route going from *Baktra* through the Pamir mountains and entering the Tarim Basin near Yarkant. This solution would explain the lack of information about the mutual position of Kashgar and Yarkant.

Assuming that Ptolemy made an error in interpreting the route described by Marinus, we have recalculated the Ptolemaic positions according to scenario C (see chapter "The Problems Outlined"), that is, adopting the identification of the *Stone Tower* as Tashkurgan. The results of our recalculation are given in Fig. 14.

The recalculated position of *Sera Metropolis* now lies in the vicinity of Lanzhou. In both scenarios the longitude of *Sera* after recalculation reduces to a very reasonable value. This illuminates the true reason for Ptolemy's decision to radically reduce the distance to *Sera* to 18,100 *stadia* in comparison to 36,200 by Marinus. Our recalculations show that this distance was not an ill-founded decision but the result of diligent cartographical work, possibly based on the comparison and linking of different distance data.

Whether Maes' agents regarded Liangzhou, Ganzhou or Lanzhou as the capital of *Serike*, or whether they had in fact reached the old capital Xi'an (in Ptolemaic times, the capital was Luoyang) cannot be decided definitely. In case Ptolemy's information came partly from sources different from Maes' informants, we cannot even be sure that the location Ptolemy called *Sera Metropolis* was identical with the *Sera* of Maes' agents.

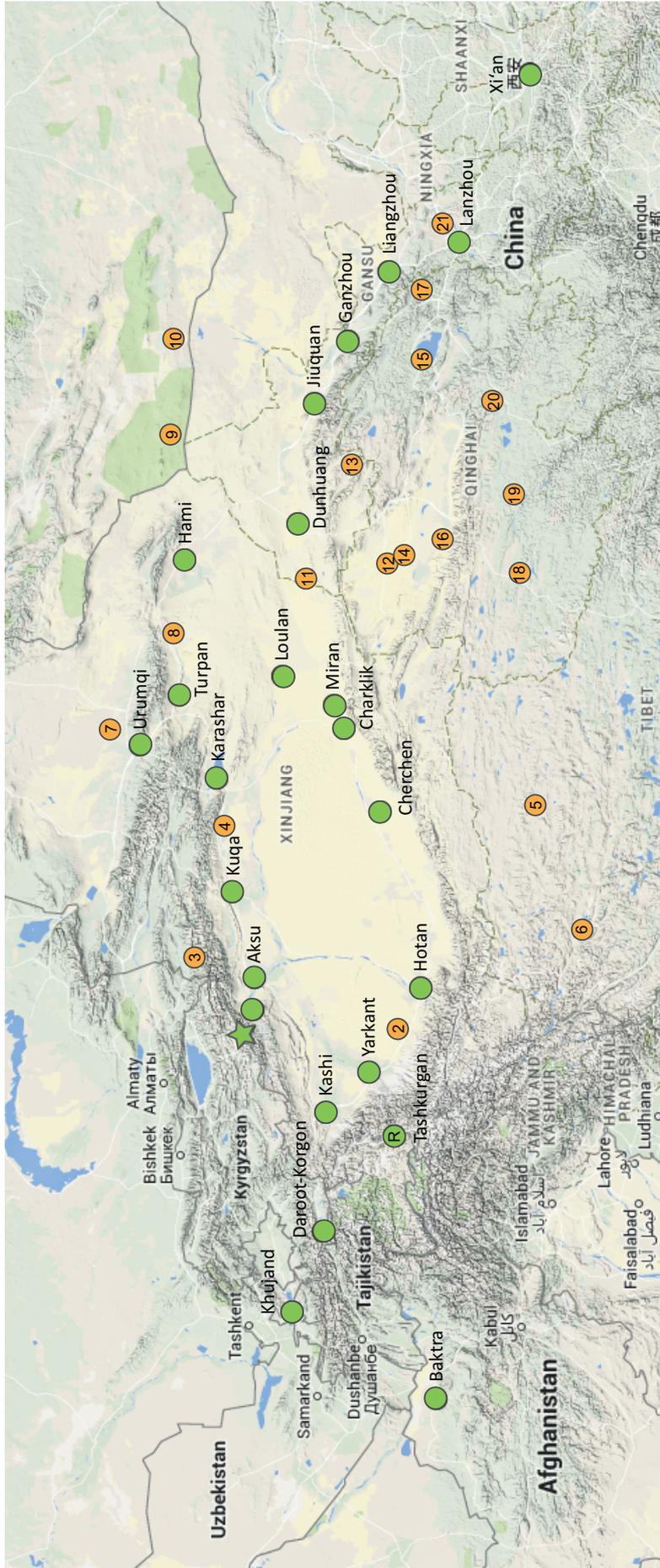


Fig. 14. Positions recalculated relative to the *Stone Tower* = Tashkurgan (marked by a circle with letter “R”).

Map data ©2015 AutoNavi, Basarsoft, Google, Map/GISrael, ORION-ME, ZENRIN.

Let us start the discussion with possible identifications in the northern part of the Tarim basin. A detailed inspection of Fig. 14 shows that, in case of the adopted identification *Stone Tower* = Tashkurgan, the recalculated positions here have a small systematic eastern longitudinal error, which may be caused by adjusting them to *hormeterion* instead of the *Stone Tower*. After reducing this error, they are firmly linked with the system of Herrmann's identification proposals lying along the Northern Route: the *Scythian Issedon* (4) as Kuqa, *Piada* (8) as Turfan, *Asmiraia* (9) as Hami and, finally, *Sera Metropolis* (21) as Liangzhou. The position of *Throana* (10) still exhibits a significant latitudinal and longitudinal error; it seems to be linked not directly with the *Stone Tower* but rather with another place along the route. We cannot subscribe to Herrmann's identification of *Damna* (7) as Karashar and *Auzakia* (3) as Aksu – they lie too far to the north. Furthermore, according to Ptolemy, *Auzakia* is situated to the west from the *Auzakia mountains* which is certainly not the case for an identification *Auzakia* with Aksu. Of course, Ptolemy could have derived the information about *Auzakia* from another *itinerarium* thus producing its wrong position on the map (the case of “overmapping” is discussed in *Appendix 2*).

Let us stress that the system of Herrmann's interpretation of locations lying along the northern route was based, in fact, on the identification of the *Scythian Issedon* with Kuga/Kuqa, justified by the exceptional size of the city in Ptolemaic times. According to *Han shu* annals,⁵² the population of the state Qiuzu, normally identified with Kuqa, exceeded 80,000 people (see item 25 in Table 1). Customizing the distances available in the *Han shu*, A. Herrmann was forced to propose the identification of *Piada* as Turfan; to explain the lack of any confluence of rivers near Turfan (which should be the case for Ptolemaic *Piada*), he attributed to Ptolemy a misunderstanding of the local river system and a mistranslation.

In fact, any identification in this region is based on the relative distances and directions to an (assumably) known location and is highly conjectural (“if B is A, then C is D”). To show how inextricably any system of identifications is connected with a preselected main identification, we also present a system resulting from our recalculations based on the identification of the *Scythian Issedon* with Aksu (see Fig. 15).

One can see that with the identification of the *Scythian Issedon* as Aksu, the recalculated position of *Piada* lies in the vicinity of Karashar, *Asmiraia* might still have been Hami,⁵³ and the *Issedon of Serike* coincides with Charklick. *Damna* now lies in the Ili Valley, on the ancient route towards the

⁵² *Han shu*, chapter 96B, 9B.

⁵³ As has already been proposed by Pulleyblank 1963: 163.

Tarim Basin going through Karashar, and *Aukazia* lies in the ancient *Wusun* territory in the vicinity of the Issyk-Kol Lake where, according to *Han shu*, should lie an old *Wusun* capital. The situational error of *Throana* becomes smaller and will be reduced even more if we recalculate its position relative to Karashar identified as *Piada*. Another possible identification for *Piada* is Quli, southwest of Kuqa; the *Shuijing zhu* 2.29a reports a residence of the “colonel of the garrison“ in the fork of confluence of the “East River”/Kuqa and the “Western Branch River” (in the vicinity of present-day Charchi).⁵⁴

From a mathematical point of view, this sequence of identifications (and, therefore, connecting distances and directions) is more convincing than the system proposed by Herrmann. Although Aksu as the *Scythian Issedon* seems to be a “second choice” (according to *Han shu*, chapter 96B, 8B, its population exceeded 24,000, which is less than that of Kuqa), the fragmentary data that Ptolemy had at his disposal need not necessarily have included information about Kuqa.

Let us stress that in contrast to Herrmann’s fuzzy approach, our recalculation depends on a single parameter – the adopted size of the Earth – or, equivalently (but less plausible), on a conversion rule for the ratio between an allegedly “alien” measuring unit and the length of the *stadion* used by Ptolemy.

To summarize, the sequence of the locations – *Stone Tower*, *hormeterion*, *Scythian Issedon*, *Piada*, *Asmiraia* and *Throana* – seem to describe a route from Tashkurgan which followed a north-south connection across the Tarim desert towards Aksu and Korla and proceeded further along the “Central Route” of *Weilue* towards Dunhuang.

⁵⁴ For discussion see also Hulsewe 1979: comm. 515.

The strange position of *Throana*/Dunhuang (assuming that this is the correct identification) relative to *Sera* can be easily explained if one assumes that Ptolemy had more than one *itinerarium* at his disposal, at least one for the Northern and one for the Southern route. The first *itinerarium* might have led only up to the border post at Dunhuang and the second started at a city which was interpreted by Ptolemy as *Sera Metropolis* and followed another (southern) route.

At first glance, one gets the impression that the positions along this southern *itinerarium* can be interpreted as places in the Qaidam Basin and Qinghai Province with *Ottorokora* lying in the vicinity of modern Golmud (see Fig. 14). In this case, the second *itinerarium* described the branches of the Qinghai Silk Road.⁵⁵ Speaking against this hypothesis is the fact that the positions of *Chaurana*⁵⁶ and *Soita*⁵⁷ will no longer support their modern indisputable identifications as Hotan and Jarkant, respectively.

Thinking along this line, we have tried to find the best possible identification of the Ptolemaic “*Sera*” with the help of our computer program. The computer solution proposes the identification of *Sera Metropolis* as Ganzhou (Zhangye). The outcome of this recalculation is given in Fig. 16.

The result is quite unexpected: our recalculation shows clearly that the *Bautisos* is not only an image which resembles the *Oichardes* with its three sources lying in three different mountain regions joining at one point⁵⁸ – it is a *double* of this river! Ptolemaic *Paliana* now lies at the position of the old Loulan kingdom (abandoned possibly after 330 B.C. due to the drying-out of Lop Nor⁵⁹ but a flourishing oasis in the desert during Ptolemy’s era), *Chaurana*’s (5) and *Soita*’s (6) positions match very well their modern identification as Hotan and Jarkant, the position of *Daxata* (17) coincides perfectly with the real position of Jiuquan in the Gansu corridor and *Drosache* (13) could be Hami.⁶⁰

The last identification is also supported by the recalculation of the position of *Soita* (yellow square in Fig. 16) relative to *Drosache*/Hami – it matches the position of Jarkant even better than the position recalculated relative to *Sera Metropolis*/Ganzhou. Quite surprisingly, *Thogara* can be a double of *Throana* – the identification of *Throana* as Dunhuang is based on the Sogdian

⁵⁵ Tong 2013: 21.

⁵⁶ Yü-T’ien in *Han shu*, see item 4 in Table 1. For discussion of different linguistic forms see, e.g. Lindegger 1993: 50.

⁵⁷ So-chü in *Han shu*, see item 7 in Table 1.

⁵⁸ This strange misinterpretation of the hydrological situation is also discussed in *Han shu* 96A, 1B.

⁵⁹ For the hydrological history of the Lop Nor see Hedin 1937.

⁶⁰ We give two positions according to formulae of *Case 1* and *2* as discussed in Tupikova/Geus 2013

pronunciation $\Delta r w' n$.⁶¹ If we assume that some parts of this *itinerarium* described also the “New Route of the North” (as described in *Weilue*), then the *Abragana* lies at this route northern to Loulan, the *Issedon of Serike* lies in the Turfan Oasis and may coincide with Jimsa (Wu-t’u of the *Han Shu*) or with Fu-k’ang (Kan-Tang). Another identification that can be considered in this context is Charkhlick as *Orosana* – the kingdom of *Loulan* was renamed after the famous historical events as *Shan-shan* (*Orosana?*) and the capital was re-established in the region of the modern Charkhlick (*Han Shu*, 96B).

Recalculated relative to *Drosache/Hami*, the position of *Orosana* (18) (yellow square in Fig. 16) matches the position of Charkhlick even better. If this sequence of identifications is correct, Charkhlick has entered the map Asia 8 twice under two different names, namely as the *Issedon of Serike* (11) and as *Orosana*.

On the background of the geographical landscape given by Ptolemy (see Fig. 11), all these identifications now make sense.

We can also bring the widely discussed location called *Ottorokora* (19) to this system of identifications – it now lies in the vicinity of the ancient city Miran (the yellow square in Fig. 16 shows its position recalculated relative to *Drosache/Hami*). Nevertheless, it cannot be excluded that the position of *Ottorokora* is related to another reference point and thus constitutes a case of “overmapping” (see Fig. A2, in Appendix 2).

⁶¹ Reichelt 1931: 48.

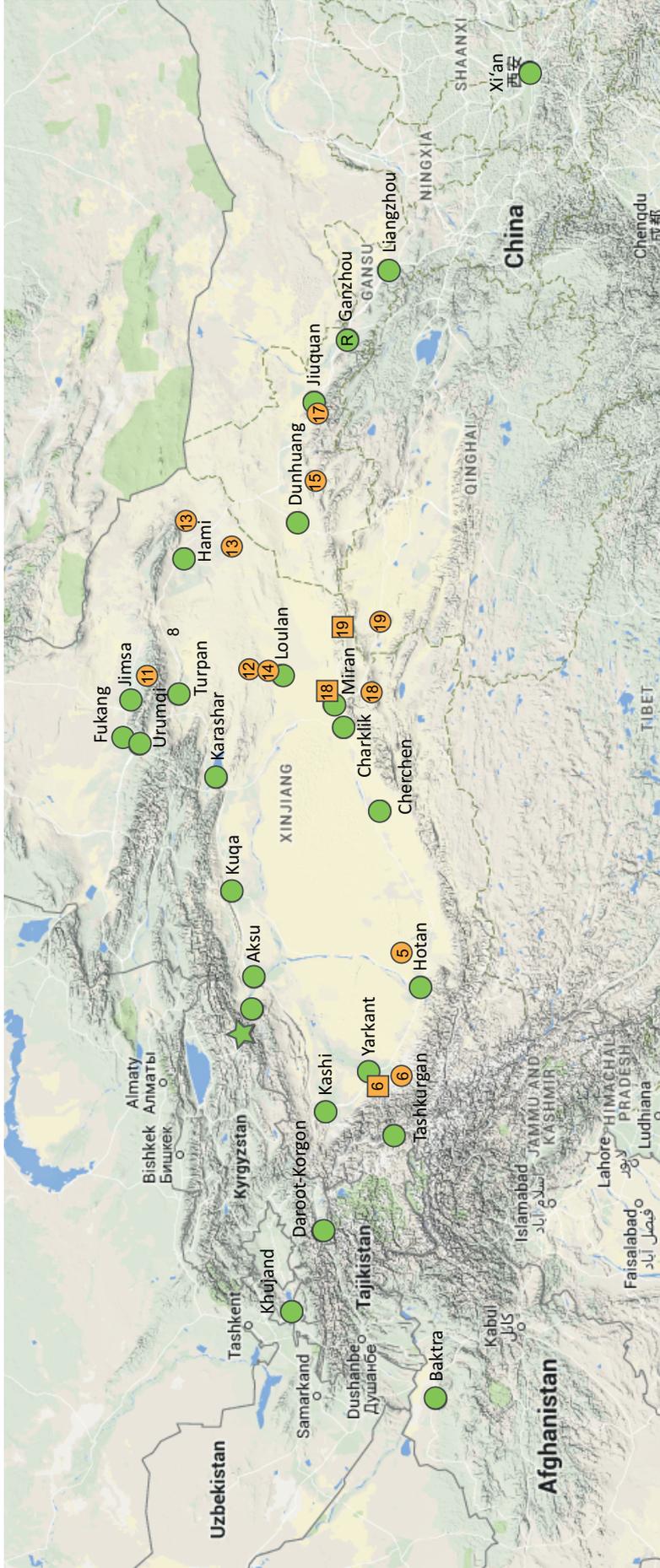


Fig. 16. Positions recalculated relative to *Sera Metropolis/Ganzhou* (yellow circles) – the reference point is marked by a green circle with “R” – and the positions recalculated relative to the identification *Drosache* = Hami (yellow squares).

Map data ©2015 AutoNavi, Basarsoft, Google, Mapa/GISrael, ORION-ME, ZENRIN.

To recalculate the landscape features we have used the following approach. First of all, Ptolemy usually gives only the utmost western and eastern parts of the mountains (in case of the Casia mountains also the utmost northern part). Of course, the definition of these extremities is highly uncertain. Although widths of mountain regions are not mentioned, one can suppose that the sources of rivers attributed by Ptolemy to a special mountain region should lie within their frames. One can further suppose that this information came from *itineraria* that Ptolemy had at his disposal in a descriptive form and was linked to a nearby mentioned place.⁶² Some typical coordinates used for a description of a mountain region can also be linked with such significant features like mountain passes. Keeping in mind the peculiarities of the problem, we have recalculated the utmost coordinates of the mountains as well as the coordinates of the river sources always relative to the nearest identified location. By that, the position of the most western part of the *Emoda* mountains is recalculated relative to *Chaurana/Hotan*; the source of *Bautisos* in these mountains relative to *Orosana/Charkhlik*; the eastern part of the *Emoda* as well as the western part of the *Ottorokora* mountains relative to *Ottorokora/Miran*; the eastern part of the *Ottorokora* mountains relative to *Sera/Ganzhou*. We would like to point out that the turning of the *Bautisos* recalculated relative to *Ottorokora/Miran* matches remarkably well with the position of the Lob Nor. According to Ptolemy, the *Bautisos* (as well as the *Oichardes*) has three sources; it reflects the Chinese idea about the origin of the Yellow River.⁶³ The silhouette of the *Casia* mountains seems to be formed according to data related to *Drosache/Hami* as well as the position of the *Thaguron* mountain. The results of our recalculation are given in Fig. 17. They match the actual landscape features in the Tarim basin surprisingly well but also underline the fact that Ptolemy had no information about the routes connecting Yarkant and Aksu with Kashgar.

As far as we know, the idea that the *Bautisos* was a double of the *Oichardes* was for the first time put forward by Antonin Wurm, a nowadays forgotten Czech scholar, whose works on Ptolemy were rarely cited even in his lifetime.⁶⁴ According to him, the confusion came into being as a result of Ptolemy's usage of different *itineraria*, which could be supported by a profound difference between the concise toponyms of Iranian and Indian origins and "the broad periphrases or even descriptions within the working-sphere of Marin-

⁶² Distances to mountains are absent in the annals of the Han Dynasty.

⁶³ For details, see e.g. Lindegger 1993: 82—84.

⁶⁴ We would like to thank Dr. Dmitriy A. Shcheglov for drawing our attention to this rare publication and for the fruitful discussion about the length of a *stadion* in Antiquity.

os”.⁶⁵ Wurm also suggested that Maes’ *Sera Metropolis* was wrongly identified by Ptolemy with a terminal station mentioned in another *itinerarium*.

More recently, the idea that the description of the map Asia 8 is “a whole double, that Ptolemy wrongly juxtaposed in a North-South way two descriptions of the Tarim basin as if they were two different regions” was put forward by Étienne de La Vaissière on comparative and semantical grounds.⁶⁶ Our recalculations confirm this idea mathematically and provide a newly interrelated system of identifications.

⁶⁵ Wurm 1926: 35.

⁶⁶ De La Vaissière 2009: 527–536.

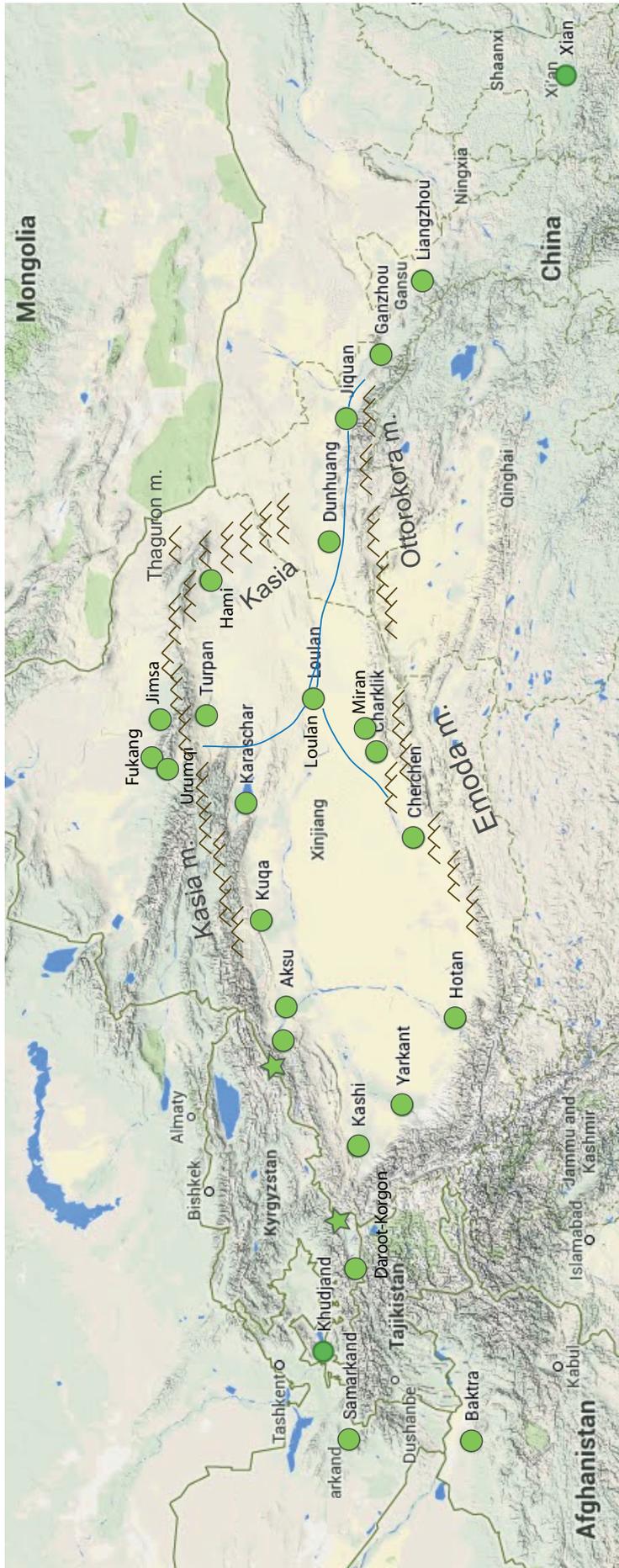


Fig. 17. Landscape features recalculated relative to the nearest identified locations.

Map data ©2015 AutoNavi, Basarsoft, Google, Mapa/GISrael, ORION-ME, ZENRIN.

Concerning the Northern Route, an important fact has to be underlined here. We have shown that the traditional identifications of the Ptolemaic positions along the Northern Silk Road are supported by a system of distances and directions measured to these locations relative to the *Stone Tower* identified as Tashkurgan. Nevertheless, due to the intricate geometry of the Ptolemaic mapping, our result does not exclude the possibility that the *Stone Tower* can also be identified with Daroot-Korgon. As argued above, Ptolemy probably used not only the *itineraria* available to Marinus, but also some additional travel records. In this case, his informants could have mentioned *another Stone Tower*, different from the *Stone Tower* of Maes' agents (this name is widely attested for old fortresses in the region under discussion), the two locations then mistakenly becoming identified in Ptolemy's catalogue.

In the same way, our approach provides a consistent set of identifications along the Southern Silk Road based on the system of distances and directions measured relative to *Sera* identified as Ganzhou (Zhangye). Whether *this Sera* is to be identified with the *Sera Metropolis* of Maes' agents remains an open question. Ptolemy's informants may have considered the first big city in the Gansu Corridor as the capital of *Seres*, and Ptolemy transmitted the distances gained from this travel account relative to its position.

What we can say for sure is that in adjusting the maps of different countries Ptolemy often succeeded in achieving spectacular precision. His results were supported by the fact that longer distances allow better to compensate deviations along the routes. How accurate Ptolemy's estimates for distances and directions towards the farthest points of the *oikoumene* were in some cases, can be demonstrated by the following example. An important Ptolemaic locality, *Palimbothra*, commonly identified as Patna, lied at the crossroads of routes to (among others) Bactria and China. Ptolemy seems to have used it as the linking reference point while adjusting the maps of these countries. The recalculated positions of *Baktra* and *Sera Metropolis* are given in Fig. 18. Whereas the position of *Baktra* coincides perfectly with the modern position of Balch, the position of *Sera* now lies in the vicinity of Xi'an. It appears that the distance and direction towards the capital of the *Seres* were known from an *itinerarium* describing the route from *Palimbothra*, and *this Sera* can – with great probability – be identified with Xi'an.

The importance of the starting (reference) point of any route cannot be overestimated due to the mathematical consequences of mapping onto a sphere of a wrong size. In particular, if we cannot decide with certainty which reference point was used by Ptolemy to calculate the position of a given locality, multiple positions of the same locality are possible.

The age-old dispute about the identification of *Sera Metropolis* – whether it is Xi'an, Liangzhou or Ganzhou - can therefore be answered in the following

way: they all can be *Sera* at the same time!

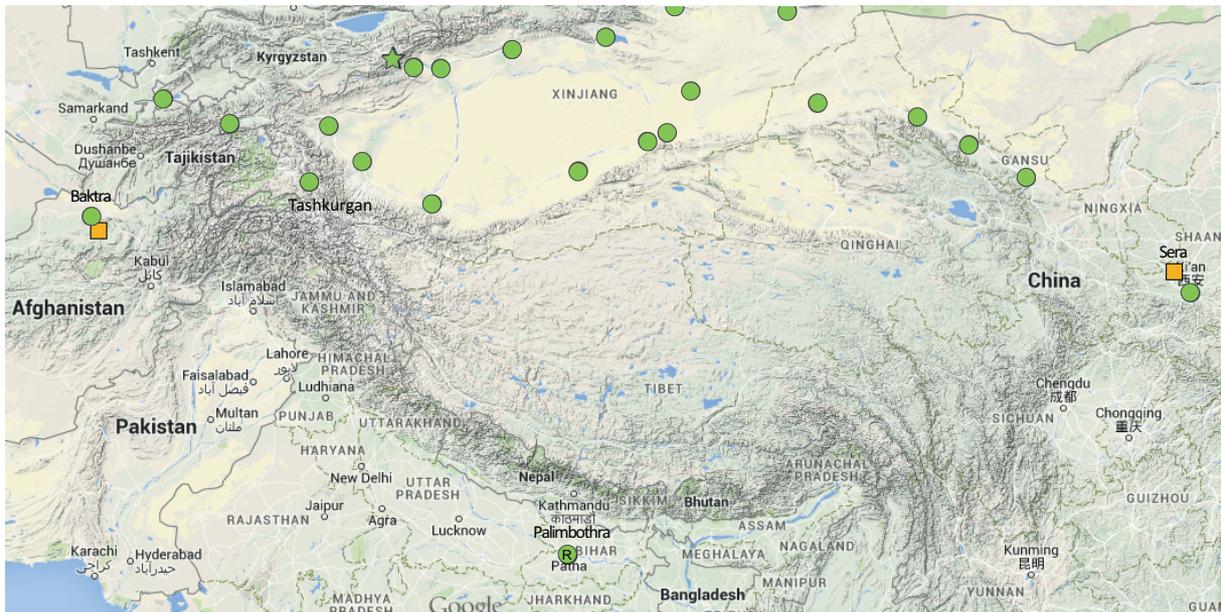


Fig. 18. Positions of *Baktra* and *Sera Metropolis* recalculated relative to *Palimbothra/Patna*.

Map data ©2015 AutoNavi, Basarsoft, Google, Mapa/GISrael, ORION-ME, ZENRIN.

Conclusion

Our discussion has shown that the application of spherical trigonometry for the recalculation of Ptolemy's coordinates opens new ways for advancing hypotheses for the identification of Ptolemaic locations. As has been argued, the most convincing origin of the systematic error in Ptolemaic coordinates lies in his assumption of a far too small circumference of the Earth. As a consequence of this error, it is not possible to reconstruct globally the Ptolemaic network of coordinates; places within one region must always be linked to at least one reference point. The method of recalculation helps to discover inconsistencies in previous attempts at systems of identification and helps putting forward new hypotheses and assessing their plausibility. The fact that this approach leads to convincing proposals for the identification of locations⁶⁷ may in turn be viewed as evidence for its adequacy.

It has become clear that Ptolemy's estimation of the size of the *oikoumene* as 180° is based on precise cartographical methods and not following a

⁶⁷ Even more clear-cut results can be gained for regions for which Ptolemy had more reliable geographical information; see Tupikova/Geus 2013, Geus/Tupikova 2013, Tupikova 2014.

preconceived symmetrical argument. Ptolemy shortened Marinus' estimate according to his own calculations by adjusting the routes coming from different *itineraria*.

As our analysis of the system of distances and directions from the *Stone Tower* to the locations along the Northern Route of the Silk Road has shown, this famous landmark of Ptolemy can with great probability be identified as Tashkurgan.

For the inner part of the Tarim Basin Ptolemy used different badly linked *itineraria*. As a result, the set of coordinates of the Tarim Basin represents in fact the same water system twice under two different names. The system of coordinates related to the *Bautisos* (Tarim) represents a part of the southern route of the Silk Road; the system of the *Oichardes* (Tarim) represents a part of the northern route.

All identifications in the Tarim are highly conditional, i.e. they depend on a preselected main identification. Therefore, one cannot claim to establish a consistent and unambiguous pattern for all Ptolemaic coordinates. One can, however, weigh the plausibility of different identifications based on a good match of a preferably large number of locations that may be linked to historical places which were clearly known to Ptolemy.

Appendix 1: The circumference of the Earth by Eratosthenes and Ptolemy

As a background to our approach to the problem of recalculation of the Ptolemaic coordinates, we will shortly present some aspects of ancient measurements of the Earth.⁶⁸

The idea to measure the Earth by using astronomical observations at places lying along the same meridian can, in all likelihood, be attributed to ancient Greek science. The first well documented name connected with this idea is that of Eratosthenes (276–194 BC).⁶⁹ An account of his method based on the sun's observation at summer solstice is handed down to us by the astronomer Cleomedes⁷⁰ who ascribes to him the famous result of 250,000 *stadia* for the Earth's circumference.⁷¹

Other authors quote the figure of 252,000 *stadia*⁷² which may have

⁶⁸ The following text is partly extracted from Tupikova 2014.

⁶⁹ A possible earlier realisation of this idea may conjecturally be attributed to Dicaearchus or Aristarchus. See, e.g. Heidel 1937: 113–121.

⁷⁰ Cleomedes, 1.10.

⁷¹ This result is mentioned also by Philoponos (*Meteor.* 1.3, p. 15 HAYDUCK [taken from Arrianus]) and Nikephoros Blemmydes (*epit. phys.* 339 [PG 142, 1277]).

⁷² Vitruvius 1.6.9; Strabo 2.5.7, 2.5.34; Pliny the Elder *nat. hist.* 2.247–8; Theophrastus of Smyrna p. 124.10–12; 127.19

been used to round the length of 1° measured along a great circle at the Earth's surface to 700 *stadia*. According to Strabo,⁷³ Hipparchus (c. 200 – c. 120 BC) mentioned Eratosthenes' measurement and accepted his result. The next method for the measurement of the Earth to be found in ancient sources is that of Posidonius (c. 135 – c. 51 BC). It is based on an observation of the star Canopus at the horizon; as Cleomedes⁷⁴ reports, Posidonius estimated the circumference of the Earth as 240,000 *stadia*.⁷⁵ Strabo in his *Geography* attached to Posidonius also the number of 180,000 *stadia*, speaking about “recent measurements of the Earth”, which “makes the Earth smallest in circumference”.⁷⁶ Strabo's remark certainly contradicts the idea of some later historians who claim that both values for the circumference of the Earth, Posidonius' of 180,000 and Eratosthenes' of 250,000, are one and the same, expressed only in different (local) variants of the unit *stadion*.⁷⁷

According to Ptolemy,⁷⁸ another method employed to calculate the circumference of the Earth was that of the observation of zenith points at two locations.⁷⁹ Subscribing to the results of these astronomical observations, Ptolemy adopted the figure of 500 *stadia* for one degree of a great circle, i.e. 180,000 *stadia* for the circumference of the Earth, claiming that this is “in accordance with the surface measurements that are generally agreed upon” (*Geogr.* 1.11.2).

Evaluating the results of measuring the circumference of the Earth one has unavoidably to tackle the question of the length of a *stadion* used by both Eratosthenes and Ptolemy.

F. Hultsch in his magisterial *Griechische und römische Metrologie* has already underlined that⁸⁰

wir im allgemein darauf verzichten müssen, aus den Stadienangaben der griechischen Schriftsteller genaue Entfernungen zu berechnen. Das στάδιον kann uns nur gelten als der konventionelle Ausdruck für 240 Schritt, und entsprechend der παράσαγγης für 7200 Schritt. Nach diesem unsichern Maßstabe wurden teils die Wegstrecken unmittelbar bestimmt, teils Parasangen und später ägyptische Schoinen und

HILLER; Gal. inst. log. 12.2; Cens. 13.2; Mart. Cap. 6.596 (cf. 609).

⁷³ *Geogr.* 1.4.1, 2.5.34.

⁷⁴ Cleomedes 1.10.

⁷⁵ See, e.g., Edelstein/Kidd 1988: Fragments 150–293.

⁷⁶ *Geogr.* 2.2.2

⁷⁷ See, e.g., Diller 1949: 7–9.

⁷⁸ *Geogr.* 1.3.1.

⁷⁹ The possible candidate stars for this method were discussed in Geus/Tupikova 2013: 71–184.

⁸⁰ Hultsch 1971 (1882): 55–56.

römische Meilen reduciert, teils endlich Entfernungen auß ungefähr mit dem Auge oder nach der Zeit abgeschätzt ... In dieses Gewirre brachte zuerst Eratosthenes einige Ordnung, indem er die mannigfachen ihm vorliegenden Angaben griechischer Schriftsteller derartig auf festes Maß reducierte, daß er das Stadion gleich 300 königlichen Ellen, mithin gleich dem vierzigsten Teile des Schoinos setzte. Allein diese Fixierung ist schwerlich zu allgemeiner Gültigkeit gelangt, und als später die geographische Forschung der Griechen mit dem römischen Wegmaß in nähere Berührung kam, da mochte wohl ein exakter Forscher wie Polibius, dessen geographische Untersuchungen uns leider nicht erhalten sind, mit umsichtiger Kritik die verschiedenen Stadienangaben vom neuen prüfen und sie mit dem römischen Wegmessungen in Einklang setzen, aber andere, die ihm folgten, warfen wieder alles durcheinander.... So hat Strabo als Normalmaß das Achtelmeilenstadion, aber daneben giebt er, wie Ideler Abhandl. 1827 S.127 nachweist, manche Ortbestimmungen nach Eratosthenes und anderen älteren Geographen, welche ein kürzeres Stadion im Sinne hatten.

Im allgemeinen also glaubten die Griechen wirklich nur *ein* Stadion als Längenmaß zu haben; es war ihnen schlechthin die Länge von 240 Schritt; allein mit welchem Grade von Genauigkeit und unter welchen Voraussetzungen dieses Maß in jedem einzelnen Falle bestimmt war, ließen sie unbeachtet.

As an example for this confusion Hultsch cites Herodotos who equalled a *schoinos* to 60 *stadia*, an error which may be ascribed to the primary usage of the notation *schoinoi* for stations for ship towing along the Nile which were of different lengths, i.e. 30, 40, 60 and even 120 *stadia*. Another example is Strabo who used as a normal *stadion* 1/8 of the Roman mile, but e.g. for the Via Appia between Rome and Aricia with the length of 16 Roman miles he calculated the route as 160 *stadia*,⁸¹ that is in relation 1:10.⁸²

Some modern scholars of ancient geography have also warned against attributing a firm modern metrical value to a *stadion*, like A. Diller who stressed that⁸³

... the Greek stade was variable and in particular instances almost always an uncertain quantity. The most problematic aspect of the ancient measurements of the earth is the length of the respective stades. Some light can be thrown on it, but the matter requires circumspection, and those who blithely convert in casual parentheses or footnotes are usually unaware of the difficulties and mistakes in their statements.

What makes the situation even more complicated, is that, according to another

⁸¹ Strab. 5.3.12, C 239.

⁸² Ideler 1827: 127.

⁸³ Diller 1949: 7–8.

er scholar, Duane Roller,⁸⁴

...there is no reason to believe that Eratosthenes always used the same length of stadion. In fact, he could not. Most of his data was based on overland or overseas distances obtained from travellers' or sailors' reports, not astronomy. Moreover, Eratosthenes used several additional forms of measurement: the *schoinos*, the sailing day, and the caravan day. And to complicate matters further, many of his distances survive only in Roman miles, which he never used ... A metrological table of late antiquity, attributed to Julian of Ascalon, calculates the Roman mile as $8 \frac{1}{4}$ of the *stadia* Eratosthenes and Strabo used, adding that the equivalent of "today" is $7 \frac{1}{2}$ *stadia*.⁸⁵ Yet Strabo himself wrote that "most" calculate eight *stadia* to the mile, but Polybios used a stadion that equalled $8 \frac{1}{3}$ to the mile.⁸⁶ Pliny used a conversion of eight *stadia*.

As Roller concludes,

... the important point is that, given these variables, and doubtless others that are unknown, it strains credulity to believe that one can determine the actual length of each and every of the many stadion distances recorded by Eratosthenes. It would have been impossible for him to have used *stadia* of the same length throughout. His distances were acquired from a variety of sources over a century, from Pytheas and the Alexander companions (if not earlier) to his own time. More importantly, they covered a wide geographical range: from eastern India to East Africa to Central Asia and northwest Europe. There is no way of determining the degree of accuracy of Eratosthenes' informants, or whether stadion distances published by these sources had already been converted from other measurements, and how accurately. One suspects that many of Eratosthenes' sources provided data in *schoinoi* ... and that he converted these, obviously at 40 *stadia* to a *schoinos*: but there is no guarantee that the original *schoinoi* were all of the same length. It is unlikely that Eratosthenes' sources gave equivalents or defined their measurements.

All the more one should expect such kind of problems in the *Geography*, whose author lived in a more globalized world and must have had much more measurement data from different parts of the world at his disposal.

Nevertheless, thanks to Eratosthenes' attempt to metricise the length of a *stadion*, one can try to interpret its length in the context of the modern metrical system. First of all, the distances in *itineraria* were measured initially, in all likelihood, not in *stadia* but in steps. Hultsch equals a step used by Eratosthenes to $2 \frac{1}{2}$ feet and estimates it as 0.656m; the length of the "Eratos-

⁸⁴ Roller 2010: 271ff. Roller's footnotes have been adopted to our style of citations.

⁸⁵ See Poseidonius, F 203 Kidd.

⁸⁶ Strab. 7.7.4, C 322.

thenian” *stadion* results then as 157.5m. With this estimate, the circumference of 252,000 *stadia* would be equal to 39,690 km and the metrical value of 1° along a great circle (e.g. equator or meridian) 700 *stadia* = 110.25 km.⁸⁷ Hipparchus also calculated 700 *stadia* per 1° and accepted the Eratosthenian result.⁸⁸

The metrical value of the *stadion* used by Ptolemy is much more debatable. Modern scholars are inclined to ascribe to Ptolemy a bigger length of the *stadion*, the majority of them 185 m per *stadion*,⁸⁹ that is, the value going back to that of 1/8 Roman mile. Accordingly, the Ptolemaic circumference of the Earth (180,000 *stadia*) would be equal to ca. 33,300 km.

Provided that Ptolemy used the same length of the *stadion* as Eratosthenes, his circumference of the Earth would attain ca. 28,305 km, that is, it would be considerably smaller than the former estimate.

In fact, the proper way to compare the relation between the *stadia* employed by both scientists is to find in ancient sources a description of the same distance of a known length mentioned both by Eratosthenes and Ptolemy.

One of the examples of Eratosthenes’ distance data is transmitted to us by Strabo⁹⁰ – this is the route between the Sacred Cap of Spain and the crossing of Euphrates by Hierapolis which was supposed to lie approximately on the parallel of Rhodes, estimated in Antiquity as 36°. The whole length of this route can be calculated from road sections, quoted by Strabo, as 30,800 *stadia*.⁹¹ If Eratosthenes had drafted a map of the *oikoumene* in a spherical coordinate system (which was, in fact, introduced not before Ptolemy), he would have calculated the angular measure of this distance at the parallel of Rhodes⁹² as $30,800 : (700 * \sin(90^\circ - 36^\circ)) = 54.39^\circ$.

In a chapter of his *Geography* entitled “The revision of the longitudinal dimension of the known world on the basis of journeys by land” (*Geogr.* 1.12), Ptolemy gives the length of the same route from the Sacred Cape in Spain to the crossing of Euphrates by Hierapolis in angular measure. Summing up his data, one obtains 69.5° for the angular length of the route.

That means that the terminus of the route, Hierapolis, would lie on Era-

⁸⁷ In comparison, the equatorial circumference of the Earth attains about 40,075 km and, accordingly, 1° is equivalent to ca. 111.3 km.

⁸⁸ Strab. 2.5.7, C 114; 2.5.34, C 132.

⁸⁹ See e.g. Stückelberger 2009: 223.

⁹⁰ Strab. 1.4.5, C 64.

⁹¹ See Geus 2012.

⁹² We have here used the ancient terminology; from a modern perspective, one cannot speak of “one degree at the parallel” of Rhodes because an angular degree is only defined along a great circle at the surface of the sphere and a parallel is not a great circle.

tosthenes' and on Ptolemy's map $69.5^\circ - 54.39^\circ = 15.11^\circ$ apart.

Because Ptolemy counted on the parallel of Rhodes 1° being equal to 400 *stadia* (to be precise, $500 \cdot \sin(90^\circ - 36^\circ) = 404$ *stadia*), the length of the route according to Ptolemy can be estimated as $69.5 \cdot 400 = 27,800$ *stadia* in "his" *stadion*. Assuming both measures to describe the same distance, we obtain a ratio of Ptolemaic to Eratosthenian *stadion*, which is, however, smaller than the ratio of 185m to 157.5m.

Let us now assume that the length of Ptolemy's *stadion* was in fact identical with the length of the Eratosthenian *stadion*. Then, on an Earth with a circumference of 252,000 *stadia*, this distance can be calculated in the angular measure as $27,800 : (700 \cdot \sin(90^\circ - 36^\circ)) = 49.1^\circ$.

The real longitudinal difference between the Sacred Cape and Hierapolis/Membidj attains ca. 47° . Consequently, the well-known overextension of the Ptolemaic map along the east-west direction decreases dramatically after recalculation of the distance to the Eratosthenian size of the Earth.⁹³

Basically, three different scenarios are possible concerning the measuring units and the assumed size of the Earth.

First scenario. Eratosthenes and Ptolemy used the same length of a *stadion* and thus had different measures for the circumference of the Earth. It attained 252,000 *stadia* (700 *stadia* per degree) by Eratosthenes and 180,000 *stadia* (500 *stadia* per degree) by Ptolemy. Due to Eratosthenes' attempt to metricise the length of his measure unit, we can attribute to his *stadion* a length of about 157.5m.

Second scenario. Ptolemy used a different length for a *stadion* than Eratosthenes but (unknowingly or without paying attention to the problem) employed distances measured in "Eratosthenian" stades in his mapping procedure.

Third scenario. Ptolemy used a different length for a *stadion* as Eratosthenes and expressed the circumference of the Earth in these units (unknown to us) as 180,000 "Ptolemaic" *stadia*.

Let us consider the first scenario. To begin with, it means that the Ptolemaic Earth is too small in comparison with the Eratosthenian size of the Earth (e.g. 28,305 km vs. 39,690 km, if one estimates the Eratosthenian *stadion* as 157.5 m). How would the mathematical consequences of this mapping procedure look like? For Ptolemy, the metrical value of one degree was 500

⁹³ Assuming both *stadia* to be of equal magnitude, one may thus conclude that the distance data became more accurate in the period between Eratosthenes and Ptolemy and that, despite all the simplifications which Ptolemy was forced to perform in his estimation of the length of this route, he was able to transform this length reliably into the arc of the great circle.

stadia; as a result, the measured distance of, e.g., 3500 *stadia*, would be represented as an arc of $3500 / 700 = \mathbf{50}$ degrees on Eratosthenes' map while as $3500 / 500 = \mathbf{70}$ degrees on Ptolemy's map, that is, it will be 40% longer.

In fact, in this case, the Ptolemaic maps should expand in *every* direction relative to every point he used as a reference or starting point for adjusting known routes. The reason why we observe primarily an extension along the east-west direction can be attributed to the usage of so-called *themelioi* ("cornerstones") in Ptolemy's system. Among the few (presumed) reliable data available to Ptolemy at his time, the rare latitudinal values of some prominent locations like Rome, Rhodes or Alexandria laid the groundwork for his mapping. To determine the longitudinal coordinates of these places, he used the terrestrial distances between these locations, found mainly in *periploi, itineraria* and other travel accounts. The framework laid out in this way could then be utilized for further local mapping. Due to his erroneously adopted size of the Earth, Ptolemy consequently obtained a bigger longitudinal difference for each pair of locations with known latitudes and known distance between them.

We will show next that, from a mathematical perspective, the second scenario is *fully equivalent* to the first scenario discussed above.

Let us assume that Ptolemy was unaware of the metrical value of the *stadion* used by Eratosthenes and considered it as being equal to his contemporary unit (e.g., to 1/8 Roman mile). Then, a distance, e.g., 700 "Eratosthenian" *stadia*, which Eratosthenes would have recalculated into degrees as 1° , will attain on the Ptolemaic map a value of $700 : 500 = 1.4^\circ$. In the same way, every distance d^E expressed in "Eratosthenian" *stadia* would be recalculated by Ptolemy in angular measure as $d^P = d^E * 700/500 = 1.4 d^E$. This is the same situation as already discussed above: the Ptolemaic map will expand in every direction with the same multiplication factor.

This case can even be reformulated in terms of an erroneously adopted size of the Earth in the following way.

Assuming that the distance $d^E = 700$ *stadia* was measured in "his" units, Ptolemy would compute "his" 1° as $700 : 1.4$, that is, equivalent to 500 (real) "Eratosthenian" *stadia*. With these 500 "Eratosthenian" *stadia* per degree, his circumference of the Earth would be $360 * 500 = 180,000$ "Eratosthenian" *stadia*.

Thus, we have shown that both situations are, from a purely mathematical standpoint, equivalent and can be treated with the same formulae. Some cartographical distortions necessarily appearing in these two scenarios will be discussed in *Appendix 2*.

It may not be obvious at first sight, but the recalculation of the positions given by Ptolemy in his *Geography* for the "Eratosthenian" size of the

Earth does not involve the use of any metrical value of the *stadion*. Spherical coordinates *per se* are dimensionless, and mathematically the procedure reduces the recalculation of coordinates given for a sphere with the circumference of 180,000 units to a sphere with the circumference of 252,000 units. The necessary formulae and results of such a recalculation were first presented in Tupikova/Geus 2013. The impressive precision of the Ptolemaic coordinates after recalculation seems to rule out the third scenario. In fact, when the length of the “Ptolemaic” *stadion* is assumed to be 185 m, so that the difference of the Ptolemaic and the Eratosthenian size of the Earth is diminished, the longitudinal extension of the Ptolemaic coordinates is not sufficiently reduced after recalculation. This is illustrated here for the map of the route towards the *Stone Tower* (Fig. 3) to which we have added the positions recalculated on the basis of the latter assumption (Fig 3A). The recalculated positions are still so far from the actual places that the positions of *Baktra*, *Marakanda* and *Alexandreia Eschate* cannot even be placed within the frame of the map (*Marakanda* lies, e.g., in the Allay valley).

The decision between the first two mathematically equivalent scenarios can only be made on the basis of historical considerations. These seem to speak for the first scenario (the same length of the *stadion* employed by both Ptolemy and Eratosthenes, and different measures for the circumference of the Earth), since Ptolemy used distance measurements from different times, including contemporary ones. Now, following the second scenario, one would have to assume that all his data were given in Eratosthenian *stadia*. Recent results of our recalculation of Ptolemaic coordinates for locations in such distant regions like *Germania Magna*,⁹⁴ for instance, show a very impressive match with their modern counterparts. The second scenario would presuppose that Ptolemy was given the distances for this far-off region expressed in Eratosthenian *stadia*, and without being unaware of their length.

Let us now show that, from a mathematical standpoint, a (supposedly) erroneous recalculation of distances transmitted in “alien” units (e.g., *parasangs*, *schoinoi*, *leugae*) into *stadia* for cartographical purposes has the same consequences as the adoption of a wrong value for the size of the Earth.

It is a well-known fact that Ptolemy, as he himself stated, adopted for his recalculations 1 *schoinos* as being equivalent to 30 *stadia*. That means that the circumference of the Earth measured in *schoinoi* would attain for him a value of $180,000 : 30 = 6000$ *schoinoi*. But if a *schoinos* was equivalent to, say, 40 *stadia*, the circumference of the Earth would measure $180,000 : 40 = 4500$ *schoinoi* (assuming that Ptolemy’s estimation of the circumference of the Earth as 180,000 *stadia* was correct). Then, the distances measured in *scho-*

⁹⁴ Geus/Tupikova 2013.

inoi on the Earth's surface (with circumference of 4500 *schoinoi*) would be converted by Ptolemy into the angular measure for the Earth's circumference equal to 6000 *schoinoi*. That would even cause a *compression* (!) of his local maps followed by cartographical problems caused by the mapping onto a sphere that is too big. This is the inverted problem of the one discussed in *Appendix 2*.

The relation 1 *schoinos* = 40 *stadia* chosen here for pure illustrative purposes is commonly ascribed to Eratosthenes due to its quotation by Pliny.⁹⁵ That the length of *schoinos* used by Ptolemy and Eratosthenes was not the same can be easily checked: even with a bigger metrical value of the *stadiion* ascribed to Ptolemy, the value of his *schoinos* would attain $30 \cdot 185 = 5550$ m in comparison with the length of the "Eratosthenian" *schoinos* which can be calculated as $40 \cdot 157.5 = 6300$ m.

Finally, we would like to draw attention to some recent statistical investigations. First, one can refer to the studies of L. Russo.⁹⁶ The main idea of his publications was to compare statistically the longitudes reported in Ptolemy's *Geography* with the actual longitude of identified positions. With a sample of about 80 identified locations, this author has obtained an estimation for the length of a *stadiion* used in Ptolemaic cartographical procedure as 155.6m – a result which is very close to the estimation of Hultsch. Second, a more refined statistical analysis with *all* identified Ptolemaic coordinates performed by K. Guckelsberger (private communication, 2013) has shown that the majority of the Ptolemaic positions was known in *stadia* in which a value of one equatorial degree was equivalent to 700 *stadia* (whatever length a *stadiion* might have had).

⁹⁵ Pliny laments in his *Natural History* (6.124) that the *schoinoi* and *parasangs* were very differently used by the previous authors and even the Persians were not consistent with them. In 11.53 he attributes to Eratosthenes the ratio 1 : 40 (parasangs : stades) but his curious wording (*patet Eratosthenis ratione*) makes it clear that it derives from his *own* calculation.

⁹⁶ Russo 2012.

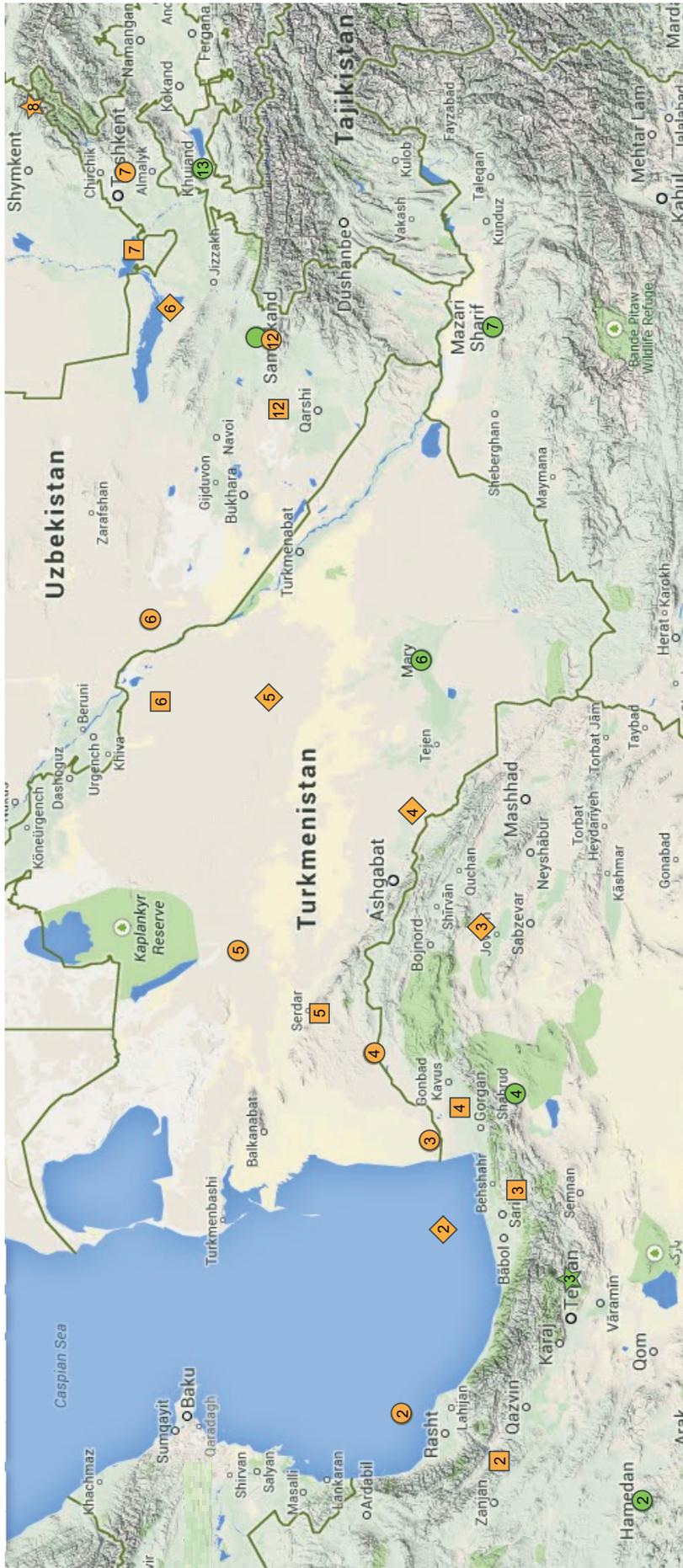


Fig. 3A. Positions recalculated assuming the Ptolemaic and “Eratosthenian” *stadia* to be the same: relative to Alexandria (yellow squares) and relative to *Hierapolis/Membidj* (yellow circles). Positions recalculated assuming the “Ptolemaic” length of the *stadiou* = 185 m, relative to Alexandria, are marked with yellow rhombi.

Map data ©2015 AutoNavi, Basarsoft, Google.

Appendix 2: Cartographical Consequences of Wrong Mapping

In order to create a system of geographical coordinates, Ptolemy used measured or rather estimated distances, directions and – clearly in very few cases – known astronomical latitudes of locations. This kind of data had to be converted into angular values for geographical latitudes and longitudes relative to a selected prime meridian. For such recalculations, the adopted circumference of the Earth is of primary importance.

Many wrong features of Ptolemy's maps can be explained as simple mathematical consequences of the erroneously adopted size of the Earth or, equivalently, of erroneously recalculated distances measured in some local units into *stadia* which he used in his mapping. We will now briefly discuss the cartographical consequences of mapping data onto a sphere with a wrong size.

As we have already shown in *Appendix 1*, every distance measured on the Earth's surface relative to some reference point will be represented on the Ptolemaic (far too small) Earth by an arc with a factor of 1.4 *bigger* as the appropriate angular distance on the Eratosthenian Earth.

Fig. A1 illustrates schematically how a map will be distorted in this case.

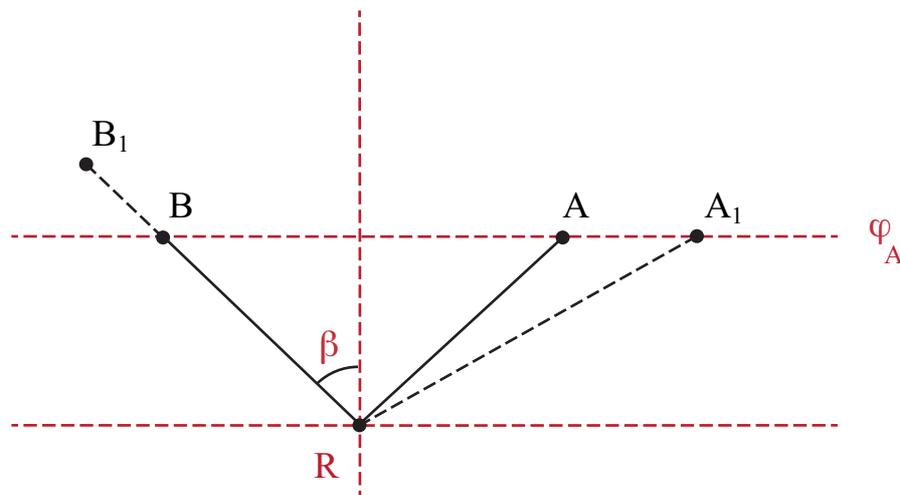


Fig. A1. Distortions on a Ptolemaic map. Point A lies at a known latitude φ_A and at a known distance s from a starting point of mapping R . The recalculated angular value of s will be bigger on the Ptolemaic map (dashed line) and the point A will be placed at the same latitude under bigger longitudinal distance to R , at point A_1 . Point B lies at a known direction (angle β) relative to R at some known distance. It will be placed in the same direction at the bigger angular distance, at point B_1 .

A side-effect of such a mapping is the apparent rotation of the line connecting points A and B relative to the meridian. Such a rotation of local maps can often be found in Ptolemaic maps (and is commonly misinterpreted by modern scholar as “misplaced” or “rotated” regional maps).

A more complicated phenomenon arises when a point, which was connected with a reference point, say, R_2 , is inserted into a local map constructed relative to another reference point R_1 . Such an “overmapping” is a necessary procedure in adjusting local maps for creating a globally coordinated map. The situation is schematically illustrated in Fig. A2.

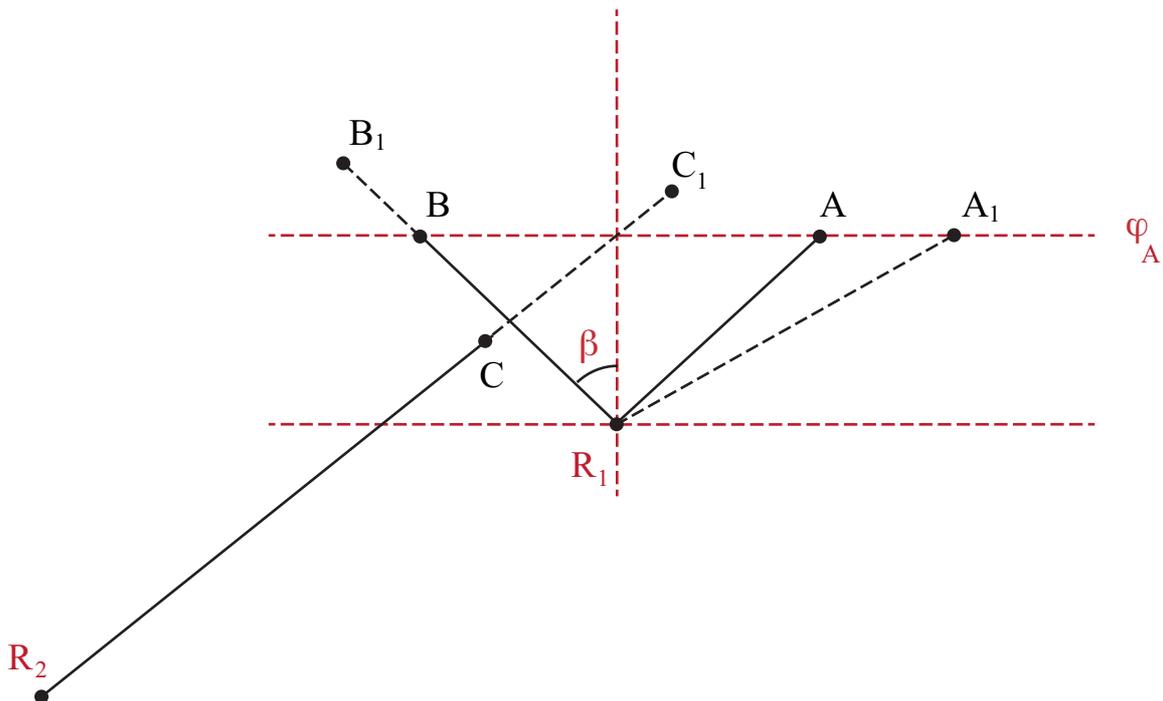


Fig. A2. “Overmapping” on a Ptolemaic map. Point C lies at a known distance s and in a known direction relative to a reference point R_2 . On the Ptolemaic map, the recalculated angular value of s will be bigger and the point C will be placed in the same direction at the bigger angular distance (point C_1).

The result of such an “overmapping” can be observed in some strange positioning of places whose mutual distances to other locations at local maps are generally distorted or skewed. An example is the position of Novaesium/Neuss which Ptolemy placed far off its true position at the wrong (“right” instead of the “left”) side of Rhine. The reason for this is due to Novaesium’s alignment – in stark contrast with other locations in *Germania Magna* which were measured relative to Colonia Agrippinensis/Cologne – to an old route connecting this military post with important cities in Gaul (Tupikova/Geus, to be pub-

lished). A problem with such “alien” locations, which are, literally, “not from this map”, consists in the ambiguity of their interpretation. One can attribute their wrong positions either to erroneously transferred distances and directions or to their alignment with other reference points. For solving this ambiguity, the historical background and the information on the presumed ancient routes are essential. Cases of “overmapping” happen very often on Ptolemaic maps due to the very nature of his information: the routes described in *itineraria* begin and end at different, sometimes very remote locations but should really be aligned on the same map.

Another “exotic” consequence of mapping to a sphere with wrong size is the “inversion” of the positions. It is shown schematically in Fig. A3.

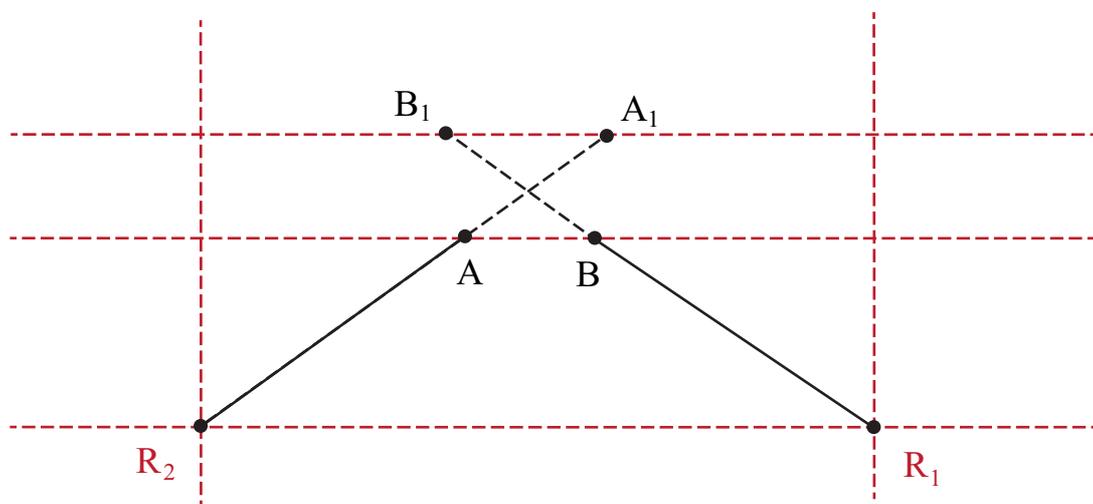


Fig. A3. “Inversion” of positions on a Ptolemaic map. The distances and directions towards points A and B are known relative to the different reference points R_2 and R_1 , respectively. On the Ptolemaic map, the recalculated angular value of distances will be bigger and the points A and B will be placed in the same direction at the bigger angular distances (dashed lines), at A_1 and B_1 , respectively.

As a result of such “inversion”, the relative positions of locations change their orientation. A good example is the positions of the German cities Noviomagus/Speyer and Bormetamagus/Worms, whose positions along the Rhine look inversed due to the mapping procedure being applied to different reference points (Tupikova/Geus, to be published).

The third mathematical consequence is the “double-mapping” of positions. It has been recognized for a long time that Ptolemy’s *Geography* contains “doubles”. Some locations which were named in different languages and/or in different *itineraria* were not always recognized as being one and the same place and hence appear twice. If they lie at different positions on Ptole-

my's maps (see Fig. A4), it is not necessarily due to wrong or inaccurate information in the *itineraria*. Even in case the distances to such positions had been known correctly relative to different reference points, the doubling of their position would be a visible mathematical consequence of mapping to a sphere of wrong size.

These three cases – overmapping, inversion and doubling – show how misleading the application of statistical methods for identifying locations on the Ptolemaic maps can be. In fact, such methods can work only for local maps constructed by Ptolemy relative to a single reference point, where the coordinates show no systematic errors. This is, of course, never the case. First, all coordinates, whose latitudes were known to Ptolemy, exhibit a different error pattern and should therefore be treated separately. Second, one can never be sure that Ptolemy calculated the coordinates on a certain map relative to one and the same reference point. Positions of locations linked with other reference points (“alien” positions) will obfuscate the statistical analysis, *per se* providing erroneous numerical parameters for recalculation of the coordinates given on a map. What is worse, the “alien” positions cannot be “improved” in such a process of recalculation. This can even give rise to wrong identifications.

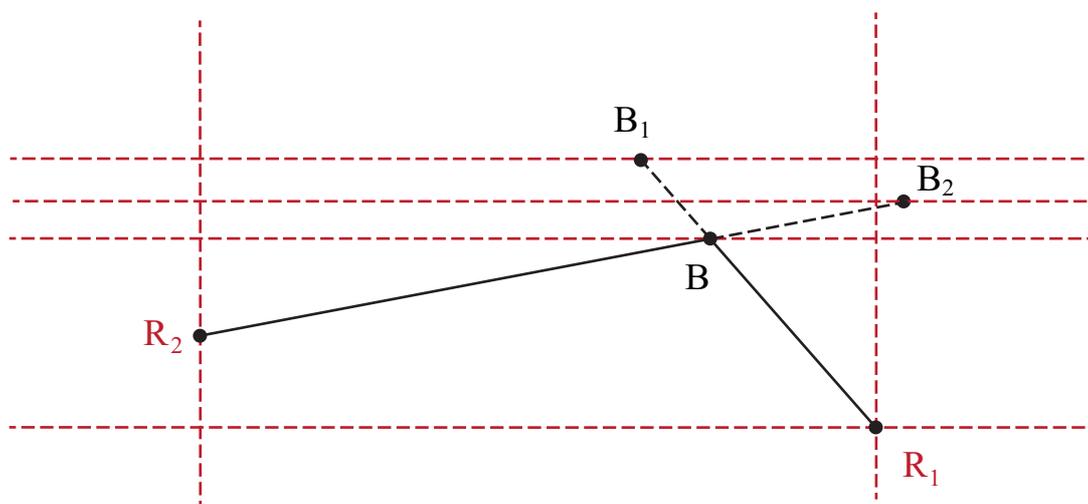


Fig A4. “Double” on a Ptolemaic map. The distances and directions towards point B, which was not recognized as one and the same location, were known relative to different reference points R_2 and R_1 . On the Ptolemaic map, the recalculated angular value of distances will be bigger and the point B will be placed in the same direction at the bigger angular distance (dashed lines) at B_1 and B_2 , respectively.

For example, statistical studies based on modern geodetic methods delivered the following identifications for the rivers in *Sarmatia* (Ptol. *geogr.* 3.5.2):

Chronos = Neman, *Rubon* = Daugava and *Turuntos* = Narva River and *Che-sinos* = Neva.⁹⁷ Recalculated to the Eratosthenian size of the Earth from a reference point attested in ancient sources, we have obtained in contrast the identifications *Chronos* = Pregolja, *Rubon* = Neman and *Turuntos* = Daugava.⁹⁸ Some have already been proposed in the literature,⁹⁹ but without any mathematical confirmation.

It is therefore clear that Ptolemy's maps can be recalculated only locally, relative to a chosen point. In other words: we are able to ascertain only which positions were linked in his local maps and how accurately the mutual distances and the directions of the routes between these locations were known. But we can never reconstruct one global Ptolemaic map.¹⁰⁰ Further, owing to the extension of Ptolemaic maps relative to every reference point, the position of the Greenwich Meridian is different relative to all the locations given on the maps.

The local Ptolemaic maps can be retrieved in the following way. First of all, let us take note of the fact that because both latitudes and longitudes are defined as central angles, a simple "blowing up" of a "smaller" sphere does not change the spherical, and hence the geographical, coordinates at all. Mathematically, the problem boils down to the transformation of a given set of spherical coordinates defined on the sphere with the circumference of 180,000 units to the set of coordinates on the sphere with the circumference of 252,000 units.¹⁰¹ In order to transform the spherical coordinates of locations one needs to use formulae of spherical trigonometry. One may argue that neither Ptolemy nor Eratosthenes knew or used such formulae; it is even known that Ptolemy applied simple *plane* triangles for local mapping. However, the goal of our study is not to show how one can improve on the calculation technique used by Ptolemy in his mapping procedure; rather the approach is to demonstrate how the Ptolemaic coordinates would look like had he adopted not a ratio of 500 but of 700 *stadia* per 1° as a scaling factor for his original data. Spherical trigonometry, in this case, provides an appropriate means to recalculate positions given in a spherical (geographical) coordinate system. All

⁹⁷ Marx/Kleineberg 2012: 50.

⁹⁸ Geus/Tupikova 2013.

⁹⁹ See, e.g., Stückelberger/Graßhoff 2006: 299.

¹⁰⁰ This *caveat* does not mean that a comprehensive Ptolemaic atlas with recalculated coordinates and correct identification for all his nearly 6400 places is impossible. But one has to factor in other considerations, especially historical ones, while selecting and evaluating the reference points (= *themelioi*). Such a challenge must be met with both mathematical and historical arguments.

¹⁰¹ The spherical coordinates should therefore be transformed and not just multiplied with an empirically obtained factor of 0.78 for a local region as, e. g., in Rinner 2013: 207ff.

the other methods would produce their own errors and would not allow to retrieve the original Ptolemaic data.

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