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Summary

The relationship between the determination of the circumference of the Earth and the geographical mapping performed by Ptolemy in his *Geography* is discussed. A simple transformation of the Ptolemaic coordinates to the circumference of the Earth measured by Eratosthenes, based on the assumption that the metrical values of the *stadion* used by both Ptolemy and Eratosthenes are equivalent, drastically improves the positions of the locations given in Ptolemy's catalogue at least for a great part of the *oikoumenē*. Comparing the recalculated positions of the identified localities with their actual positions, it turns out that the distances extracted by Ptolemy from ancient sources are remarkably precise. This in turn confirms the high precision of Eratosthenes's result for the circumference of the Earth. It is shown that many distortions of Ptolemy's world map can be explained as pure mathematical consequences of a mapping onto the surface of a sphere of wrong size.

Keywords: Ancient Geography, Ptolemy, Eratosthenes

1 Introduction

The ingenious idea to solve the problem of the measuring of the Earth by using astronomical observations of celestial objects at the places lying along the same meridian can, in all likelihood, be attributed to ancient Greek science. The first well documented name connected with this idea is that one of Eratosthenes (276–194 BC);[1] the description of his method is handed down to us by the astronomer Cleomedes.[2] According to him, at summer solstice the Sun stands in zenith over Syene and the shadow of a gnomon measures 1/50 of the full circle in a $\sigma\kappa\acute{\alpha}\varphi\eta$ placed in Alexandria. The distance between the two cities being known to be 5,000 *stadia*, this leads to the famous result of 250,000 *stadia* for Earth's circumference.[3] Other authors inform us that Eratosthenes' measurement was 252,000 *stadia*,[4] a slightly different figure, which may have been used to round the length of 1° measured along a great circle at the Earth's surface to 700 *stadia*. According to Strabo,[5] Hipparchus (c. 200 – c. 120 BC) mentioned Eratosthenes' measurement and accepted his result.[6] The next method for the measurement of the Earth to be found in ancient sources is that of Posidonius (c. 135 – c. 50 BC) who worked mainly on Rhodes; in a famous passage of Cleomedes[7] we are told that on the basis of the observation that the star Canopus

[1]The possible earlier realization of this idea may conjecturally be attributed to Dicaearchus or Aristarchus. See, e.g. Heidel W. A., *The Frame of the Ancient Greek Maps*, New York: American Geographical Society (Research Series, No. 20), 1937, 113–121.

[2]*Cleomedes* 1.10.

[3]This result is mentioned also by Philoponos (Meteor. 1.3, p. 15 HAYDUCK [taken from Arrianus]) and Nikephoros Blemmydes (epit. phys. 339 [PG 142, 1277]).

[4]Vitr. 1.6.9; Strab. 2.5.7, 2.5.34 ; Plin. nat. 2.247–8; Theo Smyrn. p. 124.10–12; 127.19 HILLER; Gal. inst. log. 12.2; Cens. 13.2; Mart. Cap. 6.596 (cf. 609).

[5]Geogr. 1.4.1, 2.5.34.

[6]For problems related to Eratosthenes' observations see e.g. Rawlins, D., "Eratosthenes' geodesy unravelled: was there a high-accuracy hellenistic astronomy?" (ISIS, 1982, 73, 259–265) and "The Eratosthenes-Strabo Nile Map. Is it the earliest surveying instance of spherical cartography? Did it supply the 5000 stades arc for Eratosthenes' experiment?" (*Archive for History of Exact Sciences*, 24. IX. 1982, V. 26, 3, pp 211–219.)

[7]*Cleomedes* 1.10.

can be seen just above the horizon at Rhodes and at an angle equal to 1/48 of the whole circle above the horizon at Alexandria, Posidonius concluded that the "circumference of the Earth is 240,000 *stadia*, if the distance from Rhodes and Alexandria is 5000 *stadia*; but if this distance is different, the circumference will be also proportionally different".[8] Strabo in his *Geography* also attached Posidonius' name to the new figure of 180,000 *stadia*, speaking about "recent measurements of the Earth", which "make the Earth smallest in circumference".[9] Strabo's remark is in stark contrast to the idea of some later historians who claim that both values for the circumference of the Earth, Posidonius' of 180,000 and Eratosthenes' of 250,000, are one and the same, expressed only in different (local) variants of the unit *stadion*.[10]

According to Ptolemy, another method used to calculate the circumference of the Earth was that of the observation of zenith points at two locations.[11] Chapter 3 of Book 1 of Ptolemy's *Geography* begins with the description of this method used by his predecessors for measuring the Earth:

> Using shadow-casting instruments, they observed the zenith points at the two ends of the interval, and obtained directly the arc of the meridian cut off by [the zenith points], which was [geometrically] similar to [the arc] of the journey [between the two locations]. This is because these things were set up (as we said) in a single plane (since the lines produced through the [two] ends [of the journey] to the zenith points intersect), and because the point of intersection is the common center of the circles. Hence they assumed that the fraction that the arc between the zenith points was seen to be of the circle through the [celestial] poles [i.e., the common meridian of the two locations] was the same fraction that the interval on the Earth was of the whole [Earth's] circumference.[12]

Actually, the shadow-casting instruments (Ptolemy speaks of *skiothera*) was not a convenient tool to observe the "zenith points" at a given locality. The more important problem was, however, to identify the position of the zenith over the *other* locality in order to measure the arc between both of them. The only possibility was to "mark" the position of the other zenith with a star. According to Simplicius,[13] the "ancients" observed two stars at one degree apart by the "dioptra", located the places at which these stars were in the zeniths, and measured the distance between them by hodometer. With this distance found as 500 *stadia*, they calculated the circumference of the Earth as 360 * 500 = 180,000 *stadia*. From the astronomical point of view, the restriction to exactly one degree of separation between the stars in zeniths is a serious handicap: a star culminating in the zenith at a place with latitude φ should have a declination δ equal to φ. Therefore, only pairs of stars with equatorial coordinates (φ, α) and $(\varphi + 1^\circ, \alpha)$ can be used in such a procedure (the same value of right ascension α guarantees that the stars culminate simultaneously). Assuming that such measuring had happened before Ptolemy's time, one can suggest with all probability that Ptolemy should have been aware of the existence of the such pairs of stars, that such special pairs must be visible with naked eyes and that they should be included in his catalogue. As we have already shown elsewhere,[14] only four star pairs roughly fulfilling this condition could be observable in the Mediterranean world in antiquity.[15] The best candidate seems to be a pair with the magnitude 3 (ν and χ *UMa*) with one component culminating in the zenith of Lysimachia in Ptolemaic time (but not in the time of Hipparchus). Since Ptolemy speaks, in fact, of "zenith points", one can suggest that only one star was involved in this measurement procedure - in this case, the star Pollux would be a most prominent candidate: it culminated at Ptolemy's time almost exactly at the distance of 1° from the zenith of Alexandria.

[8] See, e.g., Edelstein, L. / Kidd, I. G., *Posidonius II. The Commentary*, Cambridge, 1988, fragments 150–293.

[9] *Geogr.* 2.2.2.

[10] See e. g. Diller, A., "The Ancient Measurements of the Earth", *Isis*, 40, 1, No 119, 1949, 7–9.

[11] *Geogr.* 1.3.1

[12] English translation will follow Beggren, J. L., Jones, A., *Ptolemy's "Geography". An Annotated Translation of the Theoretical Chapters*, Princeton Univ. Press, 2000.

[13] *Commentary on Aristotle's De Caelo*, 548.27–549.10.

[14] Geus, K., Tupikova, I. , "Historische und astronomische Überlegungen zur Erdmessung des Ptolemaios", in: Geus, K., Rathmann, M. (eds), *Vermessung der Oikumene*, Berlin, Boston, 2013, 171–184.

[15] Two of these pairs culminated in zeniths over the Peloponnesus (but not in zeniths of places famed for astronomical observations), one pair (ν and τ *And*) had a component culminating in the zenith of Alexandria - but all these stars are faint objects with an apparent magnitude of c. 4.

Subscribing to the results of these arcane astronomical observations, Ptolemy adopted the figure of 500 *stadia* for one degree of a great circle, i.e. 180,000 *stadia* for the circumference of the Earth,[16] claiming that this is "in accordance with the surface measurements that are generally agreed upon".[17]

Why is the size of the Earth so important in geographical mapping performed by Ptolemy? In fact, it is due to novelty of his treatise where the equatorial coordinate system was probably for the first time consequently introduced in the map-making. For a local map made in Cartesian coordinate system, with axes labeled in metrical measure, the information about the size of the Earth is not inevitable. In the equatorial coordinate system, to the contrary, one needs the pairs of central angles (or, which is equivalent, the pairs of arcs of the great circles on the surface of a sphere) to describe the position of an object. If the information which Ptolemy used in his mapping procedure were gained from astronomical observations only, that is, from measured angles,[18] his coordinates would have the same value on the "small" Earth (with the circumference of 180,000 *stadia*) as well as on the "big" Earth (with the circumference of 252,000 *stadia*). In this sense one should also understand the quotation from Hipparchus "for it will not make much difference with respect to the celestial phenomena, whether the measurement followed is that of Eratosthenes or that given by later geographers" as quoted by Strabo in *Geogr.* II, 5.7. The problem leading to different kind of distortions arises in recalculating the distances measured on the surface of the real Earth in metrical values (*stadia*) into the angular values (degrees) used in geographical mapping of the Earth of some estimated size. For example, let us consider two locations with a mutual distance of $35,000$ *stadia* lying at equator. Provided that both Eratosthenes and Ptolemy used the same *stadion*, the longitudinal distance between both locations would be equivalent to $35,000 : 700 = 50°$ at the "Eratosthenian" map and to $35,000 : 500 = 70°$ at the Ptolemaic map. In general, *every* angular distance S measured along the arc of great circle on the "big" Earth will in this case correspond to the angular distance

$$s = S * 252,000/180,000 = s * 700/500 = 1.4\,S$$

on the surface of the "small" Earth.

It is clear, that the most part of the source data which Ptolemy had at his disposal for global mapping of the known *oikoumenē* was not a table of spherical coordinates, latitudes and longitudes, but measured or inferred distances expressed in *stadia*, *schoinoi*, *parasanges*, dayruns and other customary units which he had to convert into arc measures. In such recalculations, the adopted size of the Earth and the question whether Ptolemy used the same definition of *stadion* as Eratosthenes (and therefore the different value for the circumference of the Earth) become of primary importance. Without standardization of the metrical units in antiquity, no reliable answer can be found and the confusing data produced very different (and far from agreed upon) results.

F. Hultsch in his magisterial "Griechische und römische Metrologie"[19] has already underlined that

> wir im allgemein darauf verzichten müssen, aus den Stadienangaben der griechischen Schriftsteller genaue Entfernungen zu berechnen. Das $\sigma\tau\acute{\alpha}\delta\iota o\nu$ kann uns nur gelten als der konventionelle Ausdruck für 240 Schritt, und entsprechend der $\pi\alpha\rho\acute{\alpha}\sigma\alpha\gamma\gamma\eta\varsigma$ für 7200 Schritt. Nach diesem unsichern Maßstabe wurden teils die Wegstrecken unmittelbar bestimmt, teils Parasangen und später ägyptische Schoinen und römische Meilen reduciert, teils endlich Entfernungen auß ungefähr mit dem Auge oder nach der Zeit abgeschätzt In dieses Gewirre brachte zuerst Eratosthenes einige Ordnung, indem er die mannigfachen ihm vorliegenden Angaben griechischer Schriftsteller derartig auf festes Maß reducierte, daß er das Stadion gleich 300 königlichen Ellen, mithin gleich dem vierzigsten Teile des Schoinos setzte. Allein diese

[16] *Geogr.* 1.11.2.

[17] Nevertheless, some indications point to the fact that Ptolemy initially used in his *Almagest* the Eratosthenian estimation of the circumference of the Earth. See especially Schnabel, P., "Die Entstehungsgeschichte des kartographischen Weltbildes des Klaudios Ptolemaios", in *Sitzungsberichte der Preussischen Akademie der Wissenschaft*, Verlag der Akademie der Wissenschaften, Berlin, 1930, pp. 214–250.

[18] Longitudes can be considered measured both in angles and in time units due to equivalency $24^h = 360°$ originated from the rotation of the Earth.

[19] Hultsch F., *Griechische und römische Metrologie*, Akademische Druck- u. Verlagsanstalt, Graz, Austria, 1971 (reprint of 1882), pp. 5–56.

> Fixierung ist schwerlich zu allgemeiner Gültigkeit gelangt, und als später die geographische Forschung der Griechen mit dem römischen Wegmaß in nähere Berührung kam, da mochte wohl ein exakter Forscher wie Polibius, dessen geographische Untersuchungen uns leider nicht erhalten sind, mit umsichtiger Kritik die verschiedenen Stadienangaben vom neuen prüfen und sie mit dem römischen Wegmessungen in Einklang setzen, aber andere, die ihm folgten, warfen wieder alles durcheinander. So hat Strabo als Normalmaß das Achtelmeilenstadion, aber daneben giebt er, wie Ideler Abhandl. 1827 S.127 nachweist, manche Ortbestimmungen nach Eratosthenes und anderen älteren Geographen, welche ein kürzeres Stadion im Sinne hatten.
>
> Im allgemeinen also glaubten die Griechen wirklich nur ein Stadion als Längenmaß zu haben; es war ihnen schlechthin die Länge von 240 Schritt; allein mit welchem Grade von Genauigkeit und unter welchen Voraussetzungen dieses Maß in jedem einzelnen Falle bestimmt war, ließen sie unbeachtet.

As an example for this confusion Hultsch also cites Herodotos who equalled a *schoinos* to 60 *stadia*, an error which may be ascribed to the primarily usage of the notation "*schoinoi*" for stations for ship towing along the Nile which were of different lengths, i.e. 30, 40, 60 and even 120 *stadia*.

Modern scholars of ancient geography are also warned against attributing to a *stadion* a firm modern metrical value by A. Diller who stressed that[20]

> the Greek stade was variable and in particular instances almost always an uncertain quantity. The most problematic aspect of the ancient measurements of the Earth is the length of the respective stades. Some light can be thrown on it, but the matter requires circumspection, and whose who blithely convert in casual parentheses or footnotes are usually unaware of the difficulties and mistakes in their statements.

What makes the situation even more complicated, is that, according to another scholar,[21]

> ... there is no reason to believe that Eratosthenes always used the same length of stadion. In fact, he could not. Most of his data was based on overland or overseas distances obtained from travellers or sailors reports, not astronomy. Moreover, Eratosthenes used several additional forms of measurement: the schoinos, the sailing day, and the caravan day. And to complicate matters further, many of his distances survive only in Roman miles, which he never used ... A metrological table of late antiquity, attributed to Julian of Ascalon, calculates the Roman mile as 8 $\frac{1}{4}$ of the stadia Eratosthenes and Strabo used, adding that the equivalent of "today" is 7 $\frac{1}{2}$ stadia.[22] Yet Strabo himself wrote that "most" calculate eight stadia to the mile, but Polybios used a stadion that equalled 8 $\frac{1}{3}$ to the mile.[23] Pliny used a conversion of eight stadia.

As Roller concludes,

> the important point is that, given these variables, and doubtless others that are unknown, it strains credulity to believe that one can determine the actual length of each and every of the many stadion distances recorded by Eratosthenes. It would have been impossible for him to have used stadia of the same length throughout. His distances were acquired from a variety of sources over a century, from Pytheas and the Alexander companions (if not earlier) to his own time. More importantly, they covered a wide geographical range: from eastern India to East Africa to Central Asia and northwest Europe. There is no way of determining the degree of accuracy of Eratosthenes informants, or whether stadion distances published by these sources

[20] Diller, A., "The Ancient Measurements of the Earth", *Isis*, 40, 1, No 119, 1949, 7–8.

[21] Roller, Duane W.: *Eratosthenes "Geography". Fragments collected and translated with commentary and additional material.* Princeton; Oxford: Princeton University Press, 2010, 271ff.

[22] See Poseidonius, F 203 Kidd.

[23] Strab. 7.7.4, C 322.

> had already been converted from other measurements, and how accurately. One suspects that many of Eratosthenes sources provided data in schoinoi and that he converted these, obviously at 40 stadia to a schoinos: but there is no guarantee that the original schoinoi were all of the same length. It is unlikely that Eratosthenes sources gave equivalents or defined their measurements.

Nevertheless, thanks to Eratosthenes attempt to metricise the length of a *stadion*, one can try to interpret its length in the context of the modern metrical system. First of all, the distances in *itineraria* were measured initially, in all likelihood, not in *stadia* but in steps. Hultsch equals a step used by Eratosthenes to 2.5 feet and estimates it as 0.656m; the length of the Eratosthenian *stadion* results then as 157.5m. With this estimation, the circumference of the Earth would be equal to 39,690 km and the metrical value of 1° along a great circle (e.g. equator or meridian) 700 *stadia* = 110.25 km. Hipparchus also calculated 700 *stadia* per 1° and accepted 252,000 *stadia* for the circumference of the Earth[24] expressed in Eratosthenian *stadia*.

The metrical value of the *stadion* ascribed to Ptolemy is much more debatable. Often, his *stadion* is estimated as 185m, that is, its length is recalculated from the relation 1 Roman mile = 8 *stadia* as mentioned by Strabo.[25] Nevertheless, Strabo's relation could also be a simple recalculation law used in common practice. As Ideler has shown,[26] Strabo used throughout his texts also a value close to the value of the Eratosthenian *stadion* as well as some even smaller values. The latest studies aimed to compare statistically the longitudes reported in Ptolemy's *Geography* provide an estimation for the length of a stadium used in Ptolemaic cartographical procedure as 155.6m – the result which is very close to the estimation of the Eratosthenian stadium.[27]

Let us give an example for the inconsistency of modern interpretations of ancient metrical units. It is a well-known fact that Ptolemy, as he himself stated, adopted for his recalculations 1 *schoinos* as being equivalent to 30 *stadia*. The relation 1 *schoinos* = 40 *stadia* is commonly ascribed to Eratosthenes due to its quotation by Pliny.[28] With the lengths of a *stadion* prescribed to Ptolemy and Eratosthenes, the length of *schoinos* used by both scholars would not be the same. This can be easily checked: the value of Ptolemy's *schoinos* would attain only 30*185 = 5550 m in comparison with the length of the Eratosthenian *schoinos* which can be calculated as 40*157.5 = 6300 m.

The problem can be also tackled in a different way. Instead of speculating about the modern metrical value of a *stadion* used by ancient scholars, one can just recalculate the geographical positions given by Ptolemy in his *Geography* assuming that his definition of *stadion* coincides with the definition of *stadion* used by Eratosthenes in his estimation of the circumference of the Earth. This seems a safe guess as both geographers worked in Alexandria and drew on the same core of ancient geographical sources. In our opinion, it was also reasonable to suggest that Ptolemy, who had at hand the works of Hipparchus and had often used and cited his results, would certainly have mentioned (if it were the case) that his adopted value for the circumference of the Earth is the same as that one adopted by Hipparchus through Eratosthenes' measurement but expressed in the other *stadia*.

Mathematically, the problem reduces to a transformation of a given set of spherical coordinates

[24] Strab. 2.5.7, C 114; 2.5.34, C 132.

[25] According to the latest German edition of *Geography*, the Eratosthenes' stadium measures 157.5 m and the stadium of Ptolemy 185 m which makes the relation between the lengths of one degree on the Earth's surface used by both scholars to $(700 * 157.5) : (500 * 185) = 1.19$ instead of $700 : 500 = 1.4$ which would be the case for the equal lengths of stadium (A. Stückelberger, "Masse und Messungen", in *Klaudios Ptolemaios. Handbuch der Geographie*, Schwabe Verlag, Basel, 2009,p. 222–224). The same value of 185 m is also ascribed to Ptolemy's *stadion* in the annotated translation of *Geography* by Berggren & Jones, p. 20.

[26] 'Ideler, C. L., 'Über die Längen- und Flächenmasse der Alten", *Abhandlungen der Königlichen Akademie der Wissenschaften zu Berlin. Hist. - Phil. Abh.*, 1827, 111–128.

[27] See Russo L., "Ptolemy's longitudes and Eratosthenes measurement of the Earths circumference", *Mathematics and Mechanics of Complex Systems*, 2012, 1, 1, pp. 67–79; a more refined statistical analysis with all identified Ptolemaic coordinates performed by K. Guckelsberger (private communication) confirms the result.

[28] Plinius laments in his Natural History (6.124) that the *schoino*i and *parasangs* were very differently used by the previous authors and even the Persians were not consistent with them. In 11.53 Plinius attributes to Eratosthenes the ratio 1 : 40 (*parasangs* : *stadia*) but his curious wording (*patet Eratosthenis ratione*) makes it clear that it derives from his own calculation.

defined on the sphere with the circumference of 180,000 units to the set of coordinates on the sphere with the circumference of 252,000 units.[29] The comparison of the recalculated Ptolemaic coordinates for the "bigger" Earth with the modern coordinates of identified localities can confirm the assumption about the equality of Ptolemy's and Eratosthenes' *stadion* or provide instead an information about the relation of *stadia* used by both scholars. This approach was first proposed and realized in Tupikova & Geus (2013).[30] Our results for Mediterranean, Italy and Greece points towards the equality of *stadia* used by both scholars in these regions. As a result, many topological features of Ptolemy's world map - the extension along the east-west direction,[31] distortion in north-south direction as well as the mutual rotation of some local maps - can be explained as a simple mathematical consequence of the erroneously adopted size of the Earth in combination with usage of reliable astronomical data for a part of latitudes. These latitudinal values built up the fixed points on Ptolemy's map serving as staging points of the whole construction (*themelioi*); Chapter 4 of Book 1 in *Geography* is devoted to the necessity to give the priority to the astronomical observations over the travel records. Paradoxically enough, it were these reliable astronomical data which produces the irregular distortion of Ptolemy's world map. To understand in which the way it had happened, one should consider first some underlying mathematics.

2 Mathematical approach

2.1 Basic formulae

First of all, let us take note of the trivial fact that because both latitude and longitude are defined as central angles, a simple "blowing up" of the sphere does not change the spherical, and hence also the geographical, coordinates.

Let us assume that the distances available to Ptolemy were given in the same *stadia* that were used in the estimate of the Earth's circumference as 252,000 *stadia*. If the "Ptolemaic" *stadion* was equivalent to the Eratosthenian *stadion*, the coordinates given in Ptolemy's *Geography* should be transformed from a sphere with a circumference of 180,000 units to a sphere with a circumference of 252,000 units. If the relation of the lengths of the *stadia* used by Ptolemy and by Eratosthenes is assumed to be some numerical factor x, e.g., $x = 185/157.5$, the transformation should be performed from the coordinates given on a sphere with a circumference of $180,000 \cdot x$ units to a sphere with a circumference of 252,000 units.[32]

The first step of a recalculation of the original positions should be a restoration of Ptolemy's raw data, that is, the distances between different localities which he had at his disposal and – in some cases – the directions of the routes connecting these localities. Let us emphasize that without any information about the Earth's size, the exact geographical localization can not be unambiguously determined from the respective latitudes and the distance between two points alone. It can be shown that, depending on the Earth's size, different localities with different longitudes can be arrived at by routes of the same length.

To transform the spherical coordinates of localities from a sphere with a radius r to a sphere with a radius R (which will be called, for the sake of simplicity, the "small" and the "big" Earth, respectively) one can use formulae of spherical trigonometry. Although in all available textbooks on the subject the formulae are given for a sphere with a standard radius of 1 only, the generalization for the case of a sphere with a non-unity radius and the transformation between spheres of different radii is easy to perform.[33]

[29] The spherical coordinates should be transformed and not just multiplied with an empirically gained factor 0.78 for a local region as, e. g., in Rinner, E. *Zur Genese der Orstkoordinaten Kleinasiens in der "Geographie" des Klaudios Ptolemaios*, Bern Studies in the History and Philosophy of Science, 2013, p. 207ff.

[30] Tupikova I., Geus, K. , *The Circumference of the Earth and Ptolemy's World Map*, Preprint MPIWG N 422, Berlin, 2013.

[31] The reason for the extension has already been formulated by the historian of ancient geography Henry F. Tozer in his *A History of Ancient Geography*, Cambridge Univ. Press, 1897, pp. 341–342.

[32] If we knew the modern metrical value of the Ptolemaic *stadion*, his coordinates could be immediately transformed into modern coordinates.

[33] As is usual in spherical trigonometry, the possibility of ambiguity or non-existence of every solution, as can be determined

One could argue that neither Ptolemy nor Eratosthenes knew or used such formulae; it is even known that for the local mapping Ptolemy used simple plane triangles. However, the aim of this study is not to improve the calculation technique used by Ptolemy in his mapping procedure; rather, the aim is to demonstrate how a set of coordinates given on a sphere of one size can be correctly transformed to coordinates on a sphere of another size and what Ptolemaic coordinates would look like if he had adopted as a scaling factor for his initial data in *stadia* not of 500 but rather of 700 *stadia* per 1°. Spherical trigonometry, in this case, provides an appropriate modern and easy to use formal mechanism to recalculate positions given in a spherical (geographical) coordinate system.

To treat the problem with the methods of spherical trigonometry, one needs first to construct a spherical triangle. If the solution is to improve on the coordinates of one location relative to another, the vertices of such a triangle can be set to be the localities themselves and the North (or South) Pole on Earth's surface, and the sides of this triangle are the arcs of the meridians going through both locations and the arc of a great circle connecting them. The great circles describe geodetics on the sphere, which means that the arc of a great circle between two points is the shortest surface-path between them – a fact probably not mathematically proven but known to Ptolemy. The geodetic lines are analogous to "straight lines" in plane geometry and Ptolemy consistently refers to great circles as "rectilinear intervals" on the Earth's surface (*Geogr.*, 1, 2–3).

In fact, one can hope that deviations from the energetically preferable geodetic routes due to topographical features or orientation problems statistically balance each other out over big distances. Only in rare cases one can get the impression of the way which Ptolemy used to recalculate the available distances between the localities into the arcs of the great circles.[34] It is clear that the precision of the final results of mapping is influenced not only by the inaccuracy of the geographical and astronomical data possessed by Ptolemy, but also by his way of processing this information.

As Ptolemy stated,[35]

> ... we think it is necessary to state clearly that the first step in the proceeding of this kind is systematic research, assembling the maximum of knowledge from the reports of people with scientific training who have toured the individual countries; and that the requiring and reporting is partly the matter of surveying, and partly a matter of astronomical observations. The surveying component is that which indicates the relative positions of localities solely through measurements of distances; the astronomical component is [that which does the same] by means of the phenomena [obtained] from astronomical sighting and shadow-casting instruments. Astronomical observation is a self-sufficient thing and less subject to error,while surveying is cruder and incomplete [without astronomical observations]. For, in the first place, in either procedure one has to assume as known the absolute direction of the interval between the two localities in question, since it is necessarily to know not merely how far this [place] is from that, but also in which direction, that is, to the north, say, or to the east or more refined direction than these. But one cannot find this out accurately without observations by means of foresaid instruments, from which the direction of the meridian line [with respect to one's horizon], and thereby the [absolute directions] of the traversed intervals, are easily demonstrated at any place and time.

To recalculate the geographical positions given by Ptolemy to a sphere of another size, one needs to consider different cases, depending on the information respectively available to Ptolemy.

Case 1

The first procedure can be applied to places with known (e.g. through astronomical observations or through relative position to a known locality) geographical latitudes. These localities were placed by

according to standard rules, should be taken into account.

[34] An overview is given Berggren & Jones, *Ptolemy's Geography. An annotated translation of the theoretical chapters*, Princeton, Oxford, 2000, *Introduction*.

[35] *Geography* 1, 2.1–2.2

Ptolemy on his map at the proper latitudinal circles but at erroneous mutual longitudinal distances. In this case, only this mutual longitudinal distance needs to be corrected.

Let us consider two localities, a lying at latitude φ_a and b lying at latitude φ_b, with longitudinal difference $\Delta\lambda_{ab}$ as given by Ptolemy for a "small" Earth (see Fig. 1, right). We will look for a transformation to the positions A and B on a "big" Earth (Fig. 1, left) which doesn't change the latitudes of the localities, so that $\varphi_a = \varphi_A$ and $\varphi_b = \varphi_B$. To recalculate the longitudinal difference, we first need to find the angular distance s measured along the great circle connecting points a and b on the "small" Earth using the spherical law of cosines:

$$\cos s = \cos(\pi/2 - \varphi_a)\cos(\pi/2 - \varphi_b) + \sin(\pi/2 - \varphi_a)\sin(\pi/2 - \varphi_b)\cos\Delta\lambda_{ab}. \tag{1}$$

The radian measure of this distance for the "big" Earth can be determined as

$$S = s * r/R. \tag{2}$$

Now, the "true" longitudinal difference $\Delta\lambda_{AB}$, that is, the value which Ptolemy would have found out if he had used Eratosthenes' estimation of the size of the Earth, can be calculated in radian measure e. g. as follows:

$$\cos\Delta\lambda_{AB} = \frac{\cos S - \cos(\pi/2 - \varphi_A)\cos(\pi/2 - \varphi_B)}{\sin(\pi/2 - \varphi_A)\sin(\pi/2 - \varphi_B)}$$

where $\varphi_a = \varphi_A$ and $\varphi_b = \varphi_B$. The value of the longitudinal difference in degrees is then given by

$$\Delta\lambda^{\circ}_{AB} = \Delta\lambda_{AB} * 180^{\circ}/\pi.$$

The formulae for this first case provide an explanation for the excessive distortion of Ptolemy's world

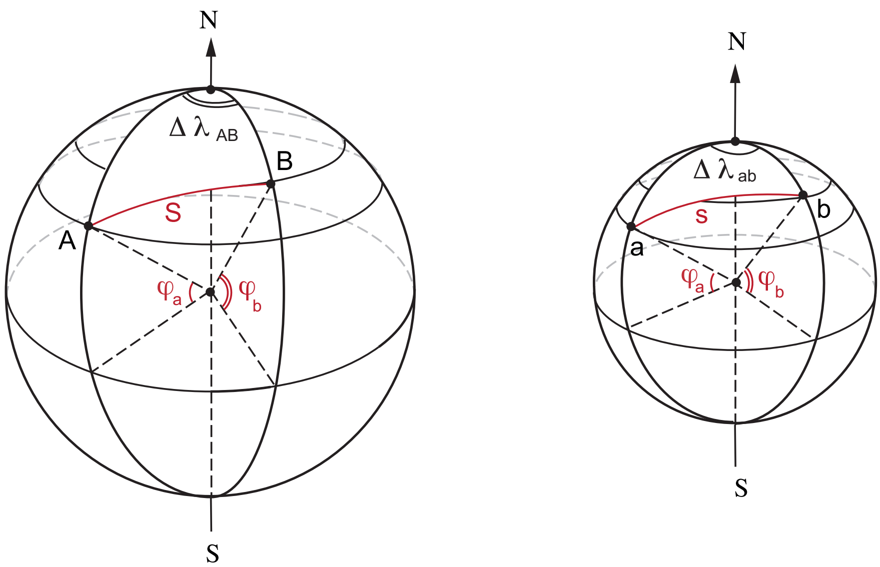


Figure 1: Recalculation of the position of a point b lying at a known latitude φ_b and at a known distance s to a starting point of mapping a with the latitude φ_a. Points A and B on the "big" Earth lie on the same latitudes as the points a and b on the "small" Earth, respectively. With the same known distance between two points, the recalculated angular value of S is smaller than s and the longitudinal distance $\Delta\lambda_{AB}$ is smaller than the longitudinal distance $\Delta\lambda_{ab}$.

map in the east-west direction (see Fig. 1).[36] The whole *oikoumenē* from the *Insulae Fortunatae* in the

[36] Throughout the diagrams, the known elements are marked with red colour.

West to the Sēra Metropolis in the Far East is equivalent to 180 degrees, too large by more than a third. Among the few reliable data which were available to Ptolemy at his time, the rare latitudinal values of some prominent locations laid the groundwork for Ptolemy's mapping. The terrestrial distances between these localities, transmitted for the most part by merchants or soldiers, were used first for determination of the longitudinal coordinates of these places. The framework obtained in this way could then be used for further local mapping. Due to the erroneously adopted size of the Earth, Ptolemy should consequently have obtained a bigger longitudinal difference for each pair of locations with known latitudes and known distance between them. Fig. 5 (points B and b, respectively) shows schematically how a map will be distorted in this case.

Case 2

The second procedure can be applied to recalculate the coordinates of a place c lying at a latitude (φ_c) which was unknown to Ptolemy (such localities obviously exhibit a significant latitudinal error). The geographical position of such a locality must have been calculated by Ptolemy only on the basis of the length of the route and the estimated direction of the route (course angle) connecting this locality with some starting point a at a known latitude φ_a. In this case, the geographical latitude of c as well as the difference in longitudes $\Delta\lambda_{ac}$ between two localities should be corrected (see Fig. 2).

To recalculate the coordinates on the "big" Earth's surface, one can proceed in the following way. First, one must restore the length of the distance and the course angle assumed by Ptolemy. The arc of the route s in radian measure can be found exactly as in the first case using Formula 1. The course angle α can be found using help of the spherical law of sines from

$$\sin\alpha = \frac{\sin\Delta\lambda_{ac}\sin(\pi/2-\varphi_c)}{\sin s}.$$

Because this value of α was, in fact, measured on the real Earth's surface, we should keep it for further calculations. With these restored data, s (expressed in *stadia*) and α, Ptolemy would have obtained his erroneous values for the latitude of the second locality φ_c and the longitudinal difference $\Delta\lambda_{ac}$. The value of the distance S in radian measure on the "big" Earth is given, once again, by Formula 2. Now, we can find the "true" latitude of the locality C using the law of cosines:

$$\cos(\pi/2-\varphi_C) = \cos(\pi/2-\varphi_a)\cos S + \sin(\pi/2-\varphi_a)\sin S\cos\alpha.$$

Here $\varphi_a = \varphi_A$.

The longitudinal difference $\Delta\lambda_{AC}$ can be calculated, e.g., with the help of the law of sines, in radian measure as

$$\sin\Delta\lambda_{AC} = \frac{\sin\alpha\sin S}{\sin(\pi/2-\varphi_C)}$$

and expressed in degrees as

$$\Delta\lambda^{\circ}_{AC} = \Delta\lambda_{AC} * 180^{\circ}/\pi.$$

The distortion of the map for this case is shown schematically in Fig. 5 (points C and c, respectively).

The geographical coordinates on the "small" Earth show the over-expansion along the east-west as well as along the north-south direction.

Case 3

A special case is that of localities lying on the same meridian (or very close to it). Lists of such cities, called *antikeimenoi poleis*,[37] circulated in antiquity since the time of the pre-geographical mapping as a means of a rough orientation between major cardinal points like important cities, ports and landmarks.

[37] Ptolemy *Geogr.* 1,4.2, 1,15.1.

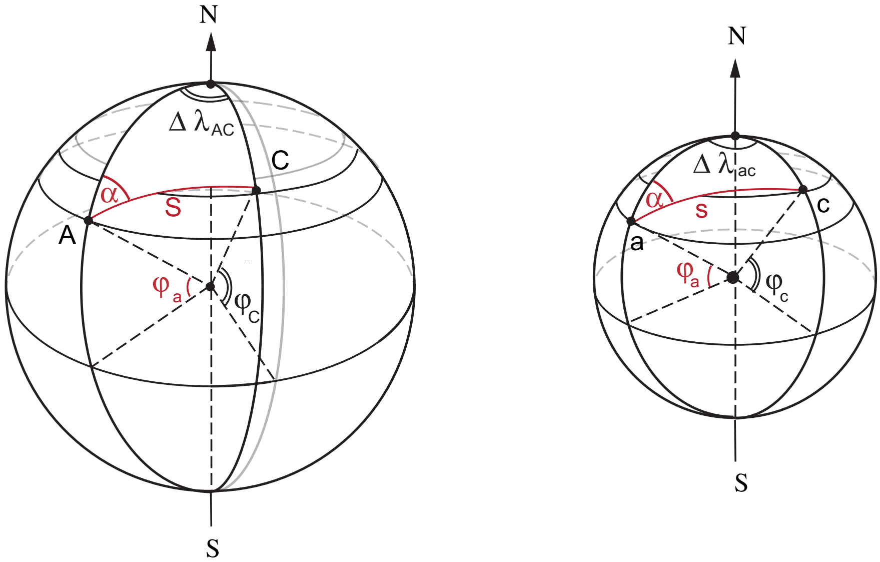


Figure 2: Recalculation of the position of a point c lying at a known distance s and in a known direction α relative to the meridian of a starting point of mapping a with latitude φ_a. Points a and A lie at the same known latitude. The recalculated position of the point C on the "big" Earth has a latitudinal value that is different from that of the point c, and lies at another latitudinal distance from the point A on the "big" Earth than the point c does relative to a on the "small" Earth.

Let point A be a reference point lying at some known latitude φ_A on the "big" Earth and point D lie at an unknown latitude at some distance S from A on the same meridian. If the distance S is expressed in *stadia* and we assume the length of a degree on the Earth surface to be 700 *stadia*, the point D will lie $S/700$ degrees to the south of A (Fig. 5, left). On the "small" Earth, where $1° = 500$ *stadia*, the appropriate point d will lie $S/500$ degrees to the south of A (Fig. 5, right). Accordingly, the latitudinal difference between a and d will attain $7/5 = 1.4$ of the latitudinal difference between A and D - that is, the point d will be placed further to the south in relation to its actual position. Hence, a point lying at a known distance to the north of A will be shifted north on the "small" Earth relative to its position on the "big" Earth. If a locality lies not exactly on the meridian of A but close to it, its position on the "small" Earth will move further to the south (or further to the north) relatively to its actual position and will also exhibit a small latitudinal displacement. This case can be treated with the formulae of Case 2.

Quite often, similar instances of unexpected latitudinal displacement can be observed on Ptolemy's world map. One such example is the notorious displacement of Carthage, ca. 4 degrees off in latitude.[38] The other example is the latitude of Kattigara, depicted by Ptolemy as lying south of the equator.[39] Both cases can easily be explained within our mathematical scheme (to be published).

It is now easy to show that this simple combination of different types of information available to Ptolemy, together with the erroneously adopted size of the Earth, automatically displaces local maps which were adjusted to a reference point of mapping in different ways relatively to each other (see Fig. 3). Instances of this kind of displacement are often observed on Ptolemy's map and they are usually considered to be a consequence of incorrect linking of the local maps. Although this was certainly partly the case, for some other maps it was just a mathematical consequence of the erroneously estimated size of the Earth.

[38] *Geogr.* 4,3.34.
[39] *Geogr.* 7,3.3.

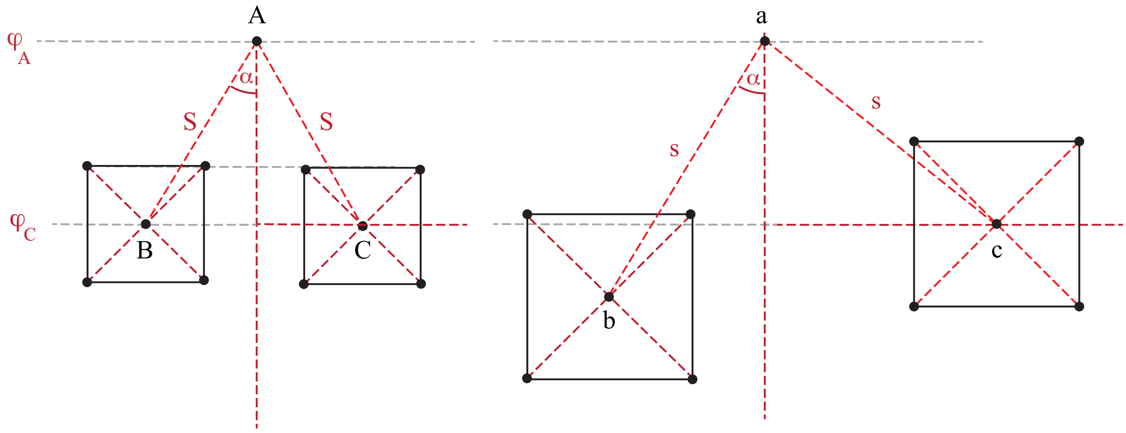


Figure 3: Relative displacement of local maps as a consequence of the erroneously adopted size of the Earth. Left: " big" Earth, right: " small" Earth. Points B and C are the reference points of the local maps (schematically represented by equal squares), lying on the same latitudinal circle. Both points and hence both dependent submaps are placed on the global map relative to a reference point A lying at distance S. The latitude of C is known *a priori* and the location is placed on the "small" Earth at point c at a known latitude, at the expense of accepting a greater longitudinal difference (Case 1). The latitude of the locality B is unknown and the location is placed on the "small" Earth at point b using information about the course angle α and the distance s to A (Case 2). The result is an apparent wholesale displacement of the local maps.

Case 4

An important one-of-a-kind case that needs to be treated separately is the position of the *Insulae Fortunatae* marking Ptolemy's zero meridian (Geogr. 1,11.1). In this case, one can assume that the longitude of the islands was supposed to be known and taken to be of supreme significance to Ptolemy; therefore, one should perform recalculation in such a way that the longitudinal distance to a reference point (e.g. Alexandria or Marseille) was kept unchanged (see Fig. 4). Because the angular distance on the "big" Earth will be shorter, the position of *Insulae Fortunatae* will lie north of the Ptolemaic position (points F and f in Fig.5). This simple idea can explain the mysterious far too southern Ptolemaic latitude of the *Insulae Fortunatae*. The solution can be calculated from the following sequence of formulae: first, the length of the distance s between the stating point a and the point lying at Ptolemy's Prime Meridian should be found exactly as in the first for the "small" Earth with Formula 1, then the route can be recalculated in angular measure (S) for the "big" Earth with Formula 2. An additional angle, let us call it β, can be found using the law of sines

$$\sin\beta = \frac{\sin\Delta\lambda_{af}\sin(\pi/2-\varphi_A)}{\sin S}$$

and then the latitude of the point F on the " big" Earth, lying on the same longitudinal circle as as the point f on the "small" Earth, can be found e.g. using the relation

$$\tan\frac{(\pi/2-\varphi_F)}{2} = \frac{\sin\frac{(\Delta\lambda_{af}+\beta)}{2}}{\sin\frac{(\Delta\lambda_{af}-\beta)}{2}}\tan\frac{(S-\pi/2+\varphi_a)}{2}.$$

Here $\varphi_a = \varphi_A$. A schematical illustration of this case is also given in the overview of the possible cases

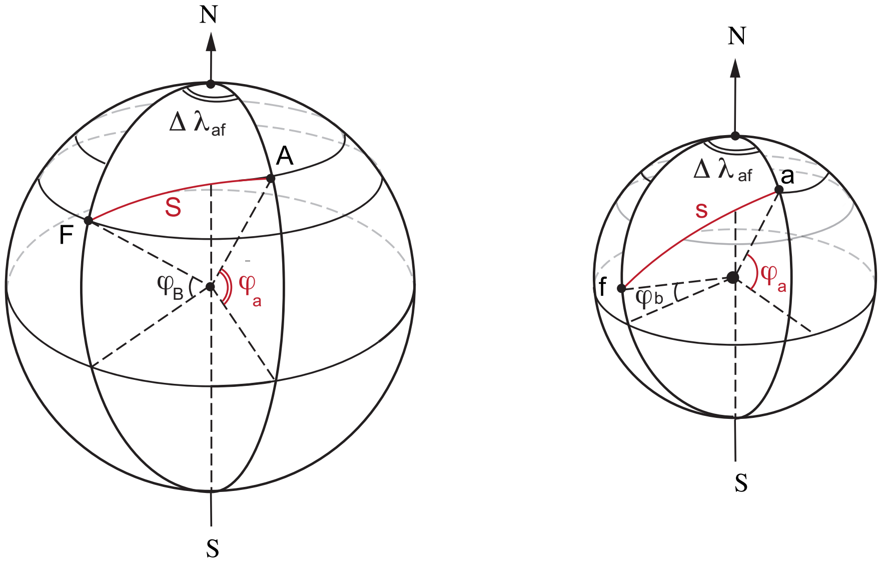


Figure 4: Recalculation of the position of Ptolemy's Prime Meridian. Points a and A are the reference points of the mapping, known *a priori* to lie at the same latitude. The *Insulae Fortunatae* (labelled f and F, respectively) are supposed to be lying at a fixed longitudinal distance $\Delta\lambda_{af}$ to the reference point of mapping. Aligning the other endpoint of the line segment of known length connecting the reference point to the islands to the Prime Meridian forces the position of the island group to slide south.

of distortion (Fig. 5).

Case 5

The case of a locality lying on the meridian of a reference point with known latitude *and* at a known distance to it was much more challenging for Ptolemy. Converting this distance into angular measure, he would not have been able to come to a conclusion consistent with the known latitude of such a locality. In this case, as a trained astronomer who would put more trust in astronomical observations than in theoretical calculations, he might have preferred to retain the known latitude of a locality on his map (Fig. 5, points E and e, respectively) and dismiss the less reliable distance measure. Such cases with "retained" latitudes can be easily spotted on Ptolemy's map; although the positioning of such localities matches their actual position very well, they are strictly speaking "not of this map" and the coordinates of the nearby localities which were not adjusted in a local map to such "alien" locations show a remarkable distorted muster. A striking example is the alignment of Syene (and therefore, Meroe) to the meridian of Alexandria. With the relative location of both cities and the latitude of Syene being very well known, Ptolemy was forced to place Syene at the proper latitudinal circle. As a result, the angular distance between Alexandria and Syene measures ca. 7.29° along the great arc connecting them and with a distance between them of 5000 *stadia*, one might get the impression that the length of a degree on the Ptolemaic map is as large as $5000 : 7.29 = 685.87$ *stadia*, very close to Eratosthenes' result. As a consequence, the following alignment of some positions along the Arabian Golf to Syene partially resembles a modern map. Nevertheless, some distances in this region (even such important ones as the distance between *Adulis* and *Aromata*) are expressed in angular measure according to the Ptolemaic low of $1° = 500$ *stadia*. Another characteristic example is the shape of Sicily, which is distorted due to his keeping of the latitude of Syracuse followed by gradual adjustment of the points on the eastern coastline at the distances recalculated by Ptolemy in angular measure according to his erroneous size of the Earth (Tupikova/Geus, to be published).

Of course, Ptolemy would have faced an insurmountable mathematical problem when adjusting ge-

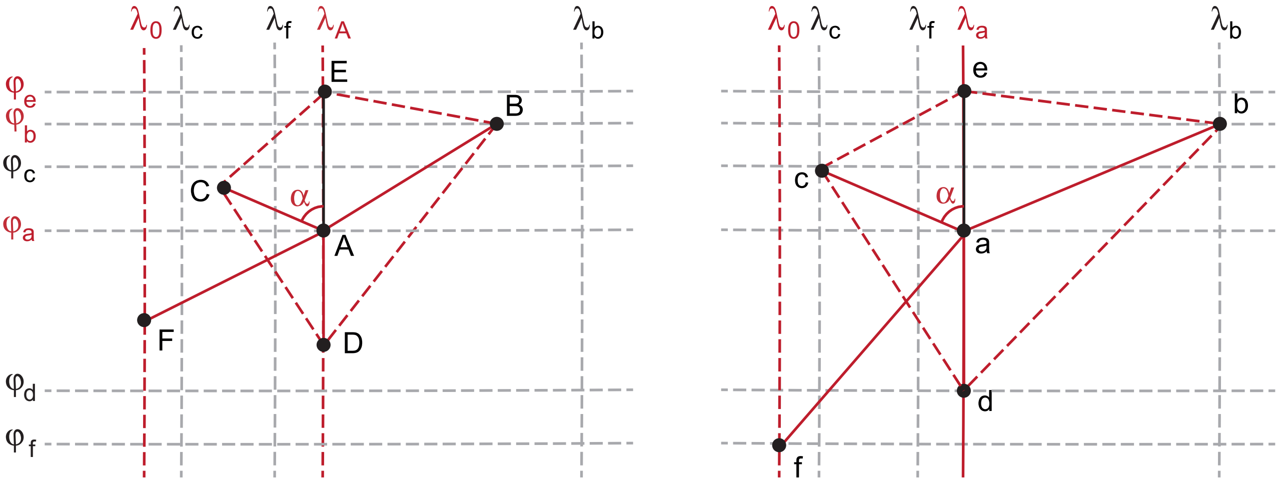


Figure 5: Schematical illustration of the possible cases of distortion in Ptolemy's world map. Left: "big" Earth. Right: "small" Earth. The point $a = A$ is the starting point of the mapping.

ographical coordinates for the case of three localities with known latitudes and known route lengths between them. In some cases, the routes would not always fit together because of the adopted erroneous value for the circumference of the Earth.

2.2 The problem of the Prime Meridian

Having adjusted Ptolemaic positions to "Eratosthenian" size of the Earth, one faces the problem of comparing the recalculated positions of the localities to their actual geographical location. Whereas Ptolemaic latitudes can be considered as being equivalent to the modern values, his longitudinal values should be corrected for the position of his Prime Meridian in relation to the Greenwich meridian. It is very important to understand that the position of the Greenwich Meridian cannot be uniquely defined in the context of Ptolemaic mapping.

Let us consider, for simplicity's sake, two known (identified) locations A and B lying at the same latitude on the "big" Earth (Fig. 6, left). The position of the *Insulae Fortunatae* is labelled with F. The longitudes of the localities A, B and F relative to the Greenwich Meridian are labelled λ_A, λ_B and λ_F, respectively. Let us assume that the localities A and B are placed by Ptolemy on the surface of the "small" Earth at points a and b_1 (Fig. 6, right) at the same (correct) longitudinal distances to the Greenwich Meridian, that is, at

$$\lambda_a = \lambda_A, \lambda_{b_1} = \lambda_B$$

and have in his catalogue the longitudes λ_a^P, $\lambda_{b_1}^P$. Then the longitude of the *Insulae Fortunatae* relative the Greenwich Meridian which can be determined from the Ptolemaic coordinates for both localities will assume the same value:

$$\lambda_F = \lambda_f = \lambda_a^P - \lambda_a = \lambda_{b_1}^P - \lambda_{b_1}.$$

On the other hand, if the position of the other locality was aligned by Ptolemy to the position of a on the basis of available information, even if this information (the length of the connecting route, the direction of travel, latitude of the location, etc.) were known precisely, the mapping onto a sphere of the wrong size would place this location at a different longitudinal distance to a, say b. As a result, the longitude of of the *Insulae Fortunatae* relative the Greenwich Meridian determined from Ptolemaic coordinates for the point a and for the point b will be different:

$$\lambda_F = \lambda_a^P - \lambda_a \neq \lambda_b^P - \lambda_b.$$

In fact, the position of Alexandria relative to the *Insulae Fortunatae* is given as 60°30′ (*Geogr.* 4,

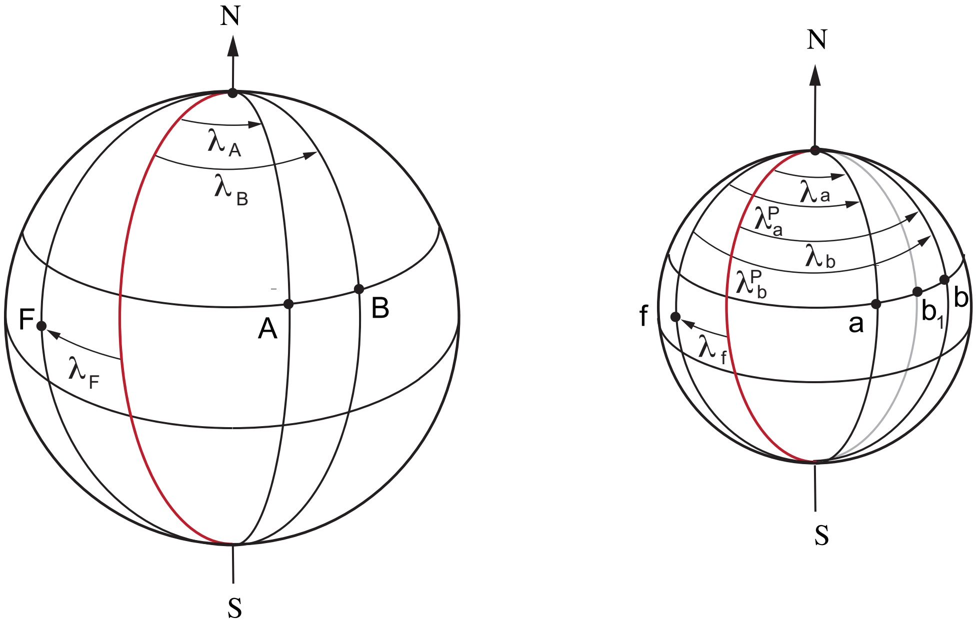


Figure 6: The problem of Ptolemy's Prime Meridian. The Greenwich Meridian is drawn in red; the positions of the *Insulae Fortunatae*/Canary Islands is labelled with F and f, respectively. The point $a = A$ is the starting point of the mapping. The position of the Greenwich Meridian relative to the *Insulae Fortunatae* is different relative to the different points after mapping on the "small" Earth's surface.

5.9); the modern longitude of Alexandria is known to be about 29° N 55′ E. Subtracting this value from Alexandria's longitude as given by Ptolemy, one can obtain the longitude of Greenwich Meridian relative to Ptolemy' Prime Meridian as 30°35′. If one attempts in the same way to recalculate the position of Greenwich Meridian with respect to the coordinates of Rome (Ptolemaic position 36°40′, modern position 12°29′), one obtains 24°11′.[40] Whichever identified location is chosen, the position of the Greenwich meridian relative to the *Insulae Fortunatae* will always come out different. The problem is not due to the poor determination of the positions in Ptolemy's time: it is due to Ptolemy's attempt to map the available distances onto a sphere of wrong size. As a result, Ptolemy's maps are locally distorted relative to *every* starting point of mapping in his source data. The maps are stretched along the east-west direction for the localities with known latitudes and along all the other possible directions in other cases. This is why the identification of the position of the Greenwich Meridian through the modern coordinates of identified localities is always complicated - it slides along the modern coordinate system and cannot be related to the Ptolemaic coordinate system *globally.* From our point of view, it makes no sense to speak of the position of the Greenwich Meridian relative to Ptolemy's zero meridian without mentioning the chosen reference point.

As a consequence, Ptolemaic maps can be recalculated and improved only *locally.*

2.3 Some refinements

The formulae of the previous chapter describe an ideal solution for transformation of a system of spherical coordinates given on a sphere of some size to spherical coordinates on a sphere of another size. The

[40]As we have already shown elsewhere (Tupikova/Geus, *The Circumference of the Earth and Ptolemy's World Map*, Preprint MPIWG N 422, Berlin, 2013.), due to the inconsistency of the longitude of Rome relative to Alexandria, coordinates recalculated relative to both localities can not be placed on the same restored map.

coordinates presented by Ptolemy in his *Geography* are far from being ideal. Not only were the distances and the relative orientation of the routes known at low precision or just estimated, even astronomical observations aimed to determine geographical latitudes were rarely performed by professional surveyors. Under such conditions a simple idea can be of great help.

Let us consider a locality which is supposed to be chosen by Ptolemy as a starting point of mapping in some local map, for instance the route to the not yet reliably identified *Stone Tower* going through *Baktra.* The known latitude of *Baktra* (Balch), 36.72305, deviates considerably from the value of 41.0° given by Ptolemy (*Geogr.* 6, 11.9). Nevertheless, the distance and the direction to the *Stone Tower* were in fact known and measured from the *real* position of *Baktra.* It seems therefore reasonable to translate the direction and the distance between both localities to the exact latitude of Balch on the Ptolemaic map, to construct a spherical triangle with this "improved" position as a vertex and then proceed to recalculate the coordinates for the "big" Earth for the purpose of identifying the site in question. This approach is illustrated in the left half of Fig. 7. The same simple idea can be applied to estimate the precision of Ptolemy's mapping even in the case of relatively well-defined latitudes which we have so far treated with the simple formulae of Case 1 (see Fig. 7, right). All the established positions in the

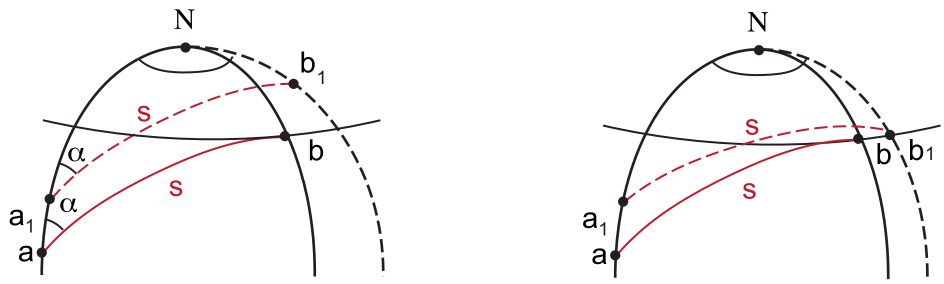


Figure 7: Translation of the Ptolemaic coordinates to the exact position of the reference point. The starting point of mapping a has a significant error in latitude; its actual position is labelled a_1. The direction to the point b and the length s of the route between a and b as perceived by Ptolemy can be translated to this real position before recalculation. Left: Case 2; right: Case 1.

following part of the text were recalculated in this way. The program we have written[41] proceeds in the following way: first, to improve the position of some locality, a suitable reference point is chosen (this choice should be justified by historical reasons). Second, a spherical triangle with vertices at the North Pole, the reference point and the locality is constructed from the spherical coordinates given by Ptolemy for both localities. Third, the distance between the localities and the direction of the route connecting them relative to the meridian of the reference point are calculated. Fourth, the distance and direction thus calculated are translated to the true position of the reference point (if available) and a new spherical triangle on the "small" Earth is constructed. Corrections appearing due to usage of these "improved" Ptolemaic positions in some regions (e.g., in the Mediterranean) are very small; they can be, of course, significant if the reference point has a big latitudinal error.

Lastly, recalculation to the "big" Earth's size is performed twice, using the formulae of Case 1 and Case 2 respectively, and both solutions are kept for further interpretation. If the actual position of the locality whose position should be recalculated is known, the better of the two solutions is retained.

3 Recalculation of Ptolemaic coordinates

The principal mathematical result of the previous chapter is the impossibility to align the position of Ptolemy's Prime Meridian, *Insulae Fortunatae*, to the position of the Greenwich Meridian *globally.* The Greenwich longitude of the Ptolemaic Prime Meridian can be established only locally, through the

[41] The complex of programs was written in the MAPLE computer algebra system.

identification of the coordinates of an identified locality at the Ptolemaic and at the modern map. The coordinates of the other localities can be recalculated relative to this identified reference point; the recalculated positions are in this way automatically referred to the Greenwich Meridian. Let us stress that the Ptolemaic localities will be placed on the modern map *at the different positions* depending on the reference point used in adjusting of the Greenwich Meridian.

The formulae of the spherical trigonometry are very sensitive relative to variation of the coordinates and an erroneously chosen reference point of mapping can produce a huge declination of recalculated positions from their proper places. Therefore, a historical background, which can provide an information about the ancient distances data, becomes a necessary part of a mathematical problem. From the other side, this sensitivity of the algorithm is of great use: it helps to sort out the positions which were in fact linked by Ptolemy on his map.

Due to amount of information in *Geography*, only a part of the results can be presented here.

First, as we have already discussed, the position of the *Insulae Fortunatae* (Ptolemy places six of them along the same meridian) slides at the Ptolemaic map towards south as a pure mathematical consequence of Ptolemy's attempt to adjust the known distances to the islands' group to the compulsory given meridian. At the "big" Earth surface, these distances (expressed in the angular measure) are shorter and the same meridian can be reached at a latitude lying northern from the Ptolemaic position. As a conceivable choice for the starting point of the route, the location of *Massilia* was chosen (corrected for the actual position of Marseille).[42] The results of recalculation performed with the formulae of Case 5 are given in Fig. 8. After recalculation, the position of the *Insulae Fortunatae* corresponds approximately to

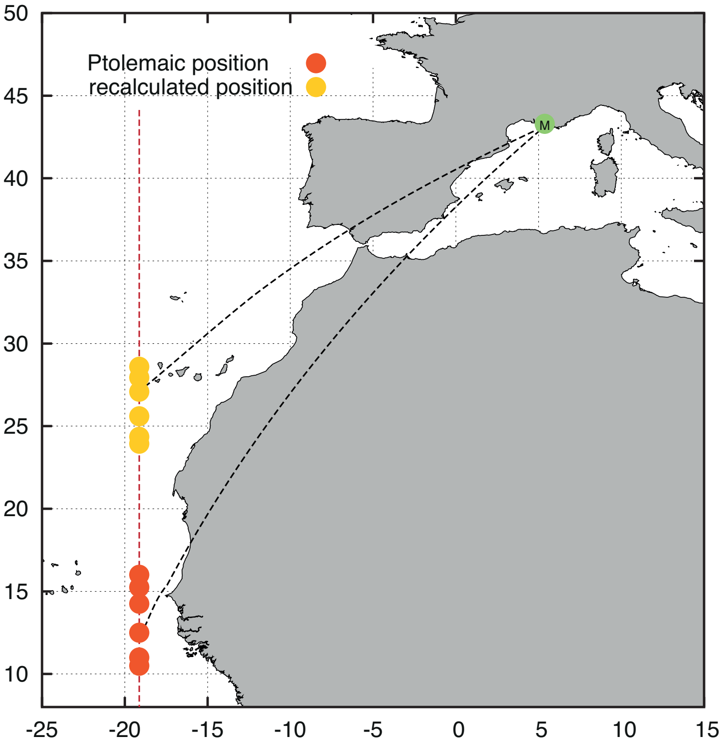


Figure 8: Recalculation of the position of the *Insulae Fortunatae* relative to *Massilia*/Marseille from the circumference of the Earth of $180,000$ units to the circumference equal to $252,000$ units.

the proper latitude of the Canary Islands instead of the Ptolemaic latitude which matches in fact better the position of the Cape Verde Islands.

[42]The coordinates of the Ptolemaic localities were taken from the new edition by Stückelberger & Graßhoff (2006).

Let us consider now Ptolemy's coordinates for the eastern part of Mediterranean. It is to be expected that, at this scale, the error in the determination of the Earth's size does manifest itself to a significant degree (although not as drastically as at the outer fringes of the *oikoumenē*); it is furthermore of great help that the positions of the historical locations are mostly reliably known and hence allow for easy verification of the results. According to the database of ancient distances compiled by K. Geus,[43] the most cited reference points are the following, in descending order: Rome, Carthage, Alexandria, Pillars of Heracles and Babylon. Because of that, we have chosen Rome (strictly speaking, the coordinates of the famous *Milliarium Aureum* at the Forum Romanum) as our reference point to recalculate the Ptolemaic positions for Spain and Gaul. Thus, the position of the Greenwich Meridian was determined through identification of the position of Rome with its modern position. Because, for the most part, the Ptolemaic latitudes in this region coincide very well with their modern counterparts, the formulae of Case 1 were used for recalculation. In order to visualise the results more clearly, only the locations of some key cities on the recalculated map are displayed in Fig. 9. A striking improvement of the coordinates in Gaul after recalculation is evident. The recalculated locations of *Burdigala*/Bordeaux and *Tolosa*/Toulouse are of a remarkable precision - it seems obvious that their positions were indeed linked to the position of Rome on the Ptolemaic map. The positions of *Lugdunum Metropolis*/Lyon, *Durocortorum*/Rheims and *Massilia*/Marseille hint at the possibility of other reference points having been used. Some of these possible reference points are easy to detect. For example, the position of

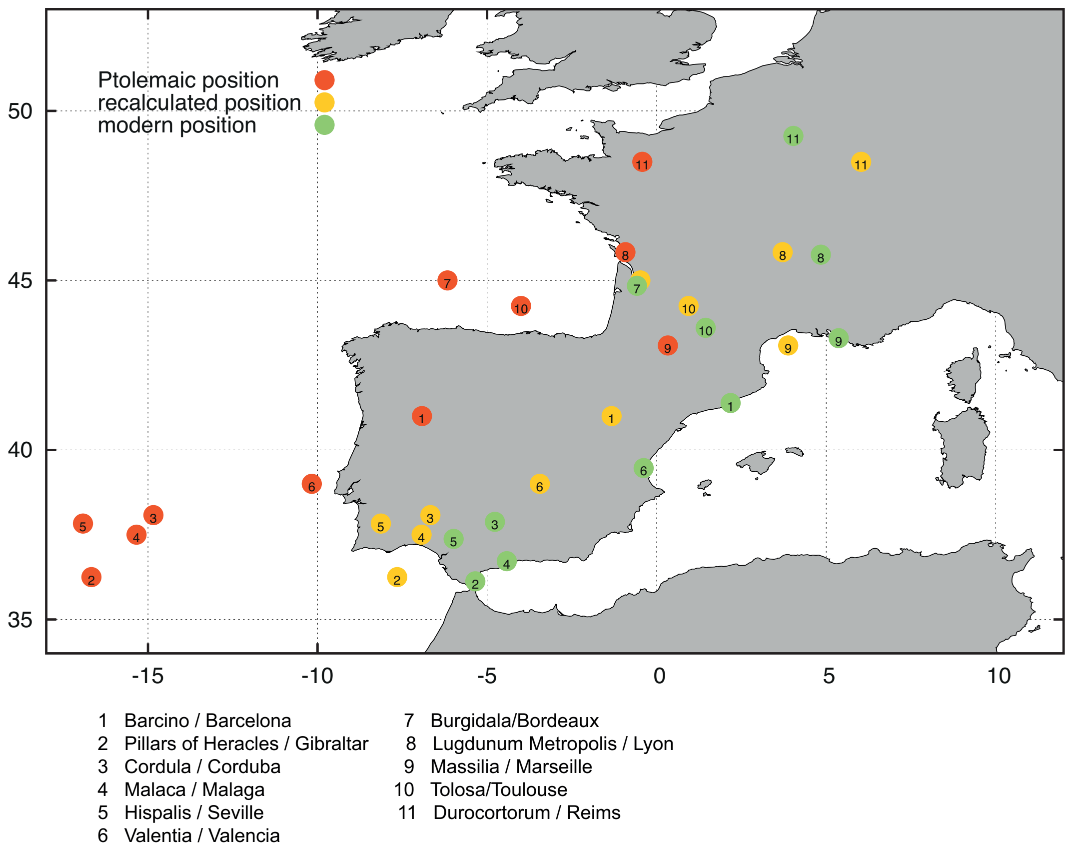
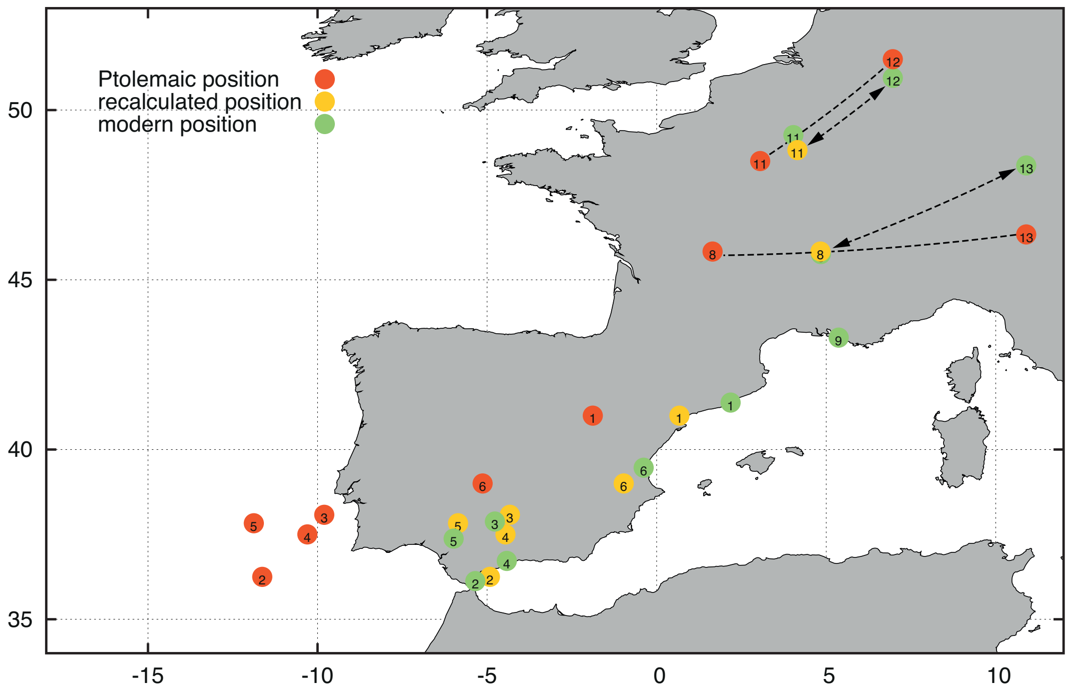


Figure 9: Recalculation of some localities in Spain in Gaul for an Earth circumference of 252,000 units. The position of the Greenwich Meridian is defined relative to Rome as a reference point.

Lugdunum Metropolis, which shows a slight longitudinal displacement, coincides perfectly with its modern

[43]This database of ancient measurements is a project carried out in the excellence cluster TOPOI (Berlin).

position after recalculating relative to the position of *Augusta Vindelicorum*/Augsburg (see Fig. 10). This recalculation was performed with the formulae of Case 1, i.e. under the assumption that the latitude of *Lugdunum Metropolis* as well as its distance to *Augusta Vindelicorum* were known. The position of *Durocortorum*/Rheims shows a latitudinal as well as the longitudinal displacement and should be recalculated with the formulae of Case 2. From historical reasoning, the position of *Durocortorum* can be linked to the position of *Colonia Agrippinensis*[44]/ Cologne and in fact, recalculation with Cologne as reference point improves the position of *Durocortorum* dramatically. In both cases, the restored Ptolemaic distances were translated before recalculation to the actual positions of the reference points. A schematical illustration of the recalculation is given in Fig. 10. The coordinates in Spain are also



1 Barcino / Barcelona
2 Pillars of Heracles / Gibraltar
3 Cordula / Corduba
4 Malaca / Malaga
5 Hispalis /Seville
6 Valentia / Valencia
8 Lugdunum Metropolis / Lyon
9 Massilia / Marseille
11 Durocortorum / Reims
12 Colonia Agrippinensis / Cologne
13 Augusta Vindelicorum /Augsburg

Figure 10: Recalculation of some localities in Spain and Gaul for an Earth circumference of 252,000 units. The position of the Greenwich Meridian is defined relative to the respective reference points of recalculation. The positions in Spain are recalculated relative to *Massilia*/Marseille using the formulae of Case 1. The position of *Lugdunum Metropolis* is recalculated relative to Augsburg, also using the formulae of the Case 1; the position of Durocortorum is recalculated relative to Cologne using the formulae of Case 2.

drastically improved but show another displacement pattern: all the recalculated coordinates lie to the west of the respective actual positions. Because a small displacement in the same direction is also seen in the coordinates of Marseille, one may hypothesise that its location served as a starting point for Ptolemy's mapping of Spain. The results of the recalculation for the same localities relative to the actual position of modern-day Marseille are given in Fig. 10. Apart from Barcino/Barcelona, which seems to be linked

[44]Full name *Colonia Claudia ara Agrippinensium.*

to another reference point, the coordinates now match the corresponding actual positions very well.[45]

A further impression of the results of the recalculation can be gained by considering the whole set of the Ptolemaic positions of identified localities in Spain. Fig. 11 (top) shows these positions adjusted to the Greenwich Meridian with respect to *Massilia* identified with the modern position of Marseille. The same positions recalculated using the circumference of the Earth given by Eratosthenes are displayed in Fig. 11, middle. One can see a significant improvement in longitudes but the well-known Ptolemaic distortion of Spain along the north-south axis is still present. It disappears, however, if one adopts the *Pillars of Heracles*/Gibraltar as the reference point of recalculation (Fig. 11, bottom).[46] In our opinion, this suggests that Ptolemy used at least two different reference points for the west part of Europe: the great part of localities were linked to Marseille, but some of the distances were known and used in the mapping procedure with respect to the *Pillars of Heracles*.

A thorough investigation of some local Ptolemaic maps has recently been applied to identify the locations given in *Geography* by statistical analysis based on the Gauss-Markov model.[47] The main idea was to choose, for every local map, a set of localities to be identified with the modern positions $(\varphi^i_{modern}, \lambda^i_{modern})$ and to compare these positions with the Ptolemaic positions $(\varphi^i_{Ptolemy}, \lambda^i_{Ptolemy})$. A system of linear equations connecting these coordinates was written as

$$\lambda^i_{Ptolemy} + v^i_\lambda = m_\lambda \lambda^i_{modern} + \lambda_0,$$
$$\varphi^i_{Ptolemy} + v^i_\varphi = m_\varphi \varphi^i_{modern} + \varphi_0.$$

In this way, the systematic differences between the modern and Ptolemaic coordinates were modelled with the scale parameters m_λ and m_φ as well as with the linear terms λ_0 and φ_0. The quantities v^i_λ and v^i_φ were considered as residual terms. After that, the subsets of locations which exhibited consistent statistical behavior without noticeable systematics in residuals were chosen. Such subsets were considered to belong to the same transformation module. With the scale parameters and the linear terms being found for every module, the modern coordinates of non-identified localities could be determined through the Ptolemaic coordinates as

$$\lambda^i_{modern} = \frac{1}{m_\lambda}\lambda^i_{Ptolemy} - \frac{1}{m_\lambda}\lambda_0,$$
$$\varphi^i_{modern} = \frac{1}{m_\varphi}\varphi^i_{Ptolemy} - \frac{1}{m_\varphi}\varphi_0.$$

Whereas the linear terms φ_0 (the authors called them "translations") of every set can be caused by an error in determination of the latitude of a reference point used by Ptolemy to map a local area, the "translation" in longitude λ_0 (which was found to be different for different subsets) is, in our opinion, to a great extent due to the problem of localization of the position of the Greenwich meridian for a selected region (see chapter 2.2). It would be mathematically reasonable to correct the Ptolemaic coordinates first for the erroneously chosen circumference of the Earth and then to proceed with statistical algorithms minimizing the residuals of the errors in the local positions. At the outermost fringes of the *oikoumenē*, where identification of the localities always involved a fair amount of guesswork, there is no sufficient input data to obtain useful results using statistical methods.

Unfortunately, the authors have taken the elongation of Ptolemy' world map as *a priori* and also seem to have attributed this main error to the use of different values for the *stadion* in ancient sources.[48]

Let us underline that the possible erroneous recalculation of local distances given in unfamiliar units into *stadia* would produce the same effect as mapping the distances onto a sphere of the wrong size, and

[45]Note that the original Ptolemaic coordinates represent different positions on the map depending on the longitude of the Ptolemy's Prime Meridian relative to the Greenwich Meridian.

[46]The results were first presented in Tupikova, I. , "Ptolemy's circumference of the Earth", in: *From Pole to Pole*, Proc. of the 26th International Cartographic Conference, ed. Buchroithner M. F., Dresden, Germany, 25 – 30 August 2013.

[47]Kleineberg et al., *Germania und die Insel Thule. Die Entschlüsselung von Ptolemaios' "Atlas der Oikumene"*, Darmstadt, 2010; Kleineberg et al., *Europa in der Geographie des Ptolemaios. Die Entschlüsselung des "Atlas der Oikumene". Zwischen Orkney, Gibraltar und den Dinariden*, Darmstadt, 2012.

[48]Kleineberg et al. *Europa in der Geographie des Ptolemaios. Die Entschlüsselung des "Atlas der Oikumene". Zwischen Orkney, Gibraltar und den Dinariden*, Darmstadt, 2012, p. 13.

the positions in such regions should be recalculated locally for a correct relation between these units in the way described in the previous chapter.

The approach discussed in this paper not only provides an epistemological explanation for the distortion observed on Ptolemy's map – in some cases, it also provides more accurate candidate positions for yet-unidentified localities. The main reason is that a choice of a single reference point (justified by historical reasons) for recalculation to the actual size of the Earth allows to avoid an averaged error introduced in the process of statistical analysis. It also makes it possible to reconstruct the internals of the Ptolemaic mapping process and to infer which pairs of mutual distances of localities were available for. An example is given in Fig. 12, where the mouths of some rivers in *Germania Magna* are shown after recalculation with the formulae of Case 2 relative to Cologne. The basis of the Ptolemaic distortion of the coastline marked with the mouths of rivers now becomes clear: the distances and the directions towards these mouths were most likely known relative to *Colonia Agrippinensis*/Cologne and the attempt to reconcile these distances with the erroneously assumed size of the Earth produced the observable result. Our recalculation using a few simple formulae places the identified rivers' mouths at their proper places (especially impressive is the precision of the positioning of the mouth of Vistula/Weichsel which marked the border of *Germania Magna* in antiquity), and helps to identify the rivers in question. The possible solution would be Stolpe or Wipper for *Viadua*, Warnow or Recknitz for *Chalusus*, Vecht for *Vidrus* and Swine/Oder for *Suebus*; it coincides with earlier proposals obtained as results of the cumbersome statistical and historical analysis. Farther to the east, in Sarmatia, the course of the "Sarmatian ocean"/Baltic sea is marked by the mouths of the not yet reliably identified rivers *Chronos*, *Rubon*, *Turuntos* and *Chesinos*.[49] As we have shown elsewhere,[50] the positions of these rivers were linked by Ptolemy with the position of the mouths of Rhine, one of which has served as a starting point for the sea expedition of Tiberius in 5 A.D.; this expedition is mentioned in the *res gestae* of Augustus and in the *Natural History* of Plinius.[51] The information about directions and distances towards the mouths of these rivers was estimated so well that it matches after recalculation to the Eratosthenian size of the Earth with a very impressive precision the positions of the mouths of Pregolja, Neman, Daugava rivers and, possibly, Salaca or Pärno rivers , the first two being common historical identifications for *Chronos* and *Rubon*, respectively. For comparison, the statistical method (Kleineberg et all, 2012, p. 50) delivered the following identifications for the rivers in Sarmatia: Chronos = Neman, Rubon = Daugava, Turuntos = Narva River and Chesinos = Neva. The reason for this strange result lies in attempt to apply the correction coefficients gained in *Germania Magna* to another region.

At the other end of the *oikoumenē*, just before the point at which the inexplicably inaccurate coordinates observed in the periphery of the Tarim basin begin, the precision of the Ptolemaic coordinates is still very high. The distances in Sogdiana and Bactria relative to Rome seem to be estimated very well and correctly transformed into lengths of spherical arcs. The results of the recalculation to the "Eratosthenian" circumference of the Earth for some locations are given in Fig. 13. Recalculation was performed with the formulae of Case 1, that is, the Ptolemaic latitudes were kept intact and only longitudinal errors produced by the erroneously assumed size of the Earth were corrected. Of remarkable precision are the positions of Kabul, Samarkand and the *Caspian Gates*. Whereas in the last case, this could be conjectured to be the result of an educated guess, for the first two cities one is tempted to explain the high-precision latitudinal values as the result of astronomical observations. The precision of *Marakanda*s coordinates is not obvious in the original Ptolemaic coordinates; its erroneous placing in Baktria even forced J. Markwart[52] to conclude that its precise latitudinal position is a pure coincidence. Detailed numerical experiments show that the position of *Marakanda* was, with a great probability, linked by Ptolemy not with the position of Rome but rather with the important position on the way towards Sogdiana and Baktria - crossing of Euphrates by *Hierapolis*/Membidj (Geogr. 5,15.13). Recalculation

[49] *Geogr.* 3,5.2.

[50] Geus, K., Tupikova, I., "Von der Rheinmündung in den Finnischen Golf . Neue Ergebnisse zur Weltkarte des Ptolemaios, zur Kenntnis der Ostsee im Altertum und zur Flottenexpedition des Tiberius im Jahre 5. n. Chr.", *Geographia Antiqua*, 2014, pp. 125–143.

[51] Plinius nat. 2, 167.

[52] Markwart, J., *Die Sogdiana des Ptolemaios*, ed. Messina, G. Rom, 1938, pp. 140–141.

of the Ptolemaic coordinates to the Eratosthenian size of the Earth with the reference point chosen at Membidj transforms the geographical coordinates of *Marakanda* to the value of 39°28′ N; 66°56′E; to compare, the actual coordinates of Samarkand are 39°39′ N, 66°58′ E. This impressive coincidence point towards an actual astronomical observation of a latitude in combination with the highly precise terrestrials measurements by the *βηματισταί* of Alexander the Great and, of course, towards a successful cartographical recalculation of the length of the route between the crossing of Euphrates and *Marakanda* into an arc of the great circle.

For two locations - *Antiocheia Margiane*/Merv and *Baktra*/Balkh - the main error for the overextension of the Ptolemaic map is eliminated but the significant latitudinal error is still extant. This profound characteristic error in latitudes was inherited by Ptolemy via Marinos from, possibly, Eratosthenes: *Baktra* and *Antiocheia Margiane* were supposed to lie at the latitude of *Hellespontes*.

The mouth of the *Oxos*/Amu Darya, which was supposed to drain into the Caspian Sea, moves towards the old riverbed of Amu (Uzboi) after the recalculation. The complicated hydrological history of the Caspian basin[53] provides reliable evidence towards this scenario in historic times. Erstwhile one of the four largest lakes in the world, the Aral Sea, was steadily shrinking and growing depending on whether the main stream of Amu Darya drained into the Caspian or towards the Aral Sea. This fact is confirmed by the discovery of archeological artefacts and ruins dated back to XIV century AD in the dried out seabed in 2001.[54] Accordingly, the level of the Caspian Sea was changing with an amplitude of about 15 meters over the course of the last three thousand years.[55] After recalculation, the position of the "center" of the *Oxeiane* (as given by Ptolemy) moves towards the eastern boundaries of the Aral sea before the beginning of the last shrinking period; it is not clear, however, how one could define a position of the "center" in a marshy depression of Syr at a time when its tributaries had not yet disappeared into the sands.

At the northern coastline of the Caspian Sea, apart from the *Rha*/Volga, only the *Daix*/Ural River seems to be reliably identified. After our recalculation, the longitudinal error is reduced but the mouths of all the rivers still show the famous significant latitudinal error. The information about these rivers was very likely gained either from older *iterinaria*[56] or from the results of the scientific expedition dispatched by Alexander the Great from the newly established *Alexandreia Eschate*; the results of recalculation relative to *Alexandreia Eschate*/Kujand are given in Fig. 14. Because of uncertainties of positions in this regions, the solutions obtained with formulae of the both basic cases are displayed at the map. Remarkably, the mouth of the *Oxos* lies now approximately at the site of the Garabogazköl lagoon and the northern coastline of the Caspian Sea looks much more realistic.

Let us recall that the *Oxeiane* is mentioned to be "built up" by one of the rivers (in fact, it was mostly Syr Darya that fed it); it might be the case that one of the northern affluents of the Caspian Sea was erroneously assumed to be a continuation of Syr Darya in this direction and appeared at the Ptolemaic map as a doublet gained from the travel reports from both western and eastern directions, once as *Iastus* and once as a misalignment of the *Iaxartus*.[57]

The whole longitudinal dimension of the Ptolemaic *oikoumenē* attains 180° demonstrating the well-known overextension of his maps. Commonly, modern scholars have explained this value assuming that Ptolemy was primarily interested in attuning the longitudinal extension of his world map to this a priori value for the sake of symmetry or for better application of his cartographic projections. To show that it was not the case, let us consider the famous position of the *Sēra Metropolis* lying according to Ptolemy at 38°35′ N and at 177°15′ east relative to his Prime Meridian (*Geogr.* 6,16.8). The route towards *Scythia on this side of the Imaon* is described by Ptolemy in the chapters entitled "On the computations that

[53]A short overview is given in *Klaudios Ptolemaios Handbuch der Geographie. Ergänzungsband.*, 2009, pp. 312–318.

[54]In the context of the INTAS program of the Instutute of History and Ethnology in Kasachstan.

[55]Aklichev, M. M. *Latentnye fenomeny kaspiyskoy depressii. Opyt analiza prirodnykh anomaliy i problem ekologicheskoy bezopasnosti*, Machachkala, Dagestan Veralg, 2000.

[56]Rapin C., "L'Incompréhensible Asie centrale de la carte de Ptolémeé: Propositions pour un décodage" , in: Bopearachchi, O.;, Bromberg, C.; Grenet, F. (eds): *Alexander' Legacy in the East: Studies in Honor of Paul Bernard = Bulletin of the Asia Institute* 12, 1998/2001, pp. 201–205.

[57]That could also explain the overestimated extension of the northern coastline of the Caspian Sea as a result of linking of the different *iterinaria.*

Marinos improperly made or the longitudinal dimension of the *oikoumenē*" and "The revision of the longitudinal dimension of the known world on the basis of journeys by land" (Geogr. 1,11–12). That is, here the primary interest of Ptolemy concerns finding out the angular distance to the most remote part of the known world. A first choice for a reference point used by Ptolemy for determination of the geographical position of the *Sēra Metropolis* seems to be the *Stone Tower* often identified with the Daroot-Korgon. After recalculation, the coordinates of the *Sēra Metropolis* attain the value of $38°26'N, 101°40'$ E[58] thus placing this city between Ganzhou/Zhangye (38°56′ N, 100°27′ E) and Liangzhou (37°52′ N, 102°43′ E), both well-known proposals for its identification. This simple recalculations show that the Ptolemaic longitudinal length of the *oikoumenē* was not an ill-founded decision but a result of a diligent cartographical work based on the comparison and linking different distance data.[59]

4 Conclusion

In regard to the size of the Earth used by Eratosthenes and Ptolemy, three scenarios are basically possible:

1. Eratosthenes and Ptolemy used the same length of a *stadion* and therefore measured differently the circumference of the Earth. It attained 252,000 *stadia* (700 *stadia* per degree) for Eratosthenes and 180,000 stadia (500 *stadia* per degree) for Ptolemy.

2. Ptolemy used a different length for a *stadion* than Eratosthenes but employed (unknowingly or without paying attention to the problem) in his mapping procedure distances measured in Eratosthenian stades.

3. Ptolemy used a different length for a *stadion* than Eratosthenes and expressed the circumference of the Earth in these (unknown to us) units as 180,000 "Ptolemaic" *stadia*.

The first scenario means, first of all, that the Ptolemaic Earth is too small in comparison with the Eratosthenian Earth (i.e. 28,305 km vs. 39,690 km, if one estimates the Eratosthenian *stadion* as 157.5 m). The recalculation of spherical coordinates given on a sphere of one size to a sphere of another size is simple from the mathematical point of view, but requires some experience in the subject. Surprisingly, it has never, to my knowledge, been applied to recalculate the geographical positions given by Ptolemy in his *Geography* for Eratosthenes's circumference of the Earth.

The results of such a recalculation show that if Ptolemy had adopted Eratosthenes's figure, the majority of his positions would have had coordinates which match their modern counterparts remarkably well. As a consequence, one can confirm first the very high precision of Eratosthenes's result for the circumference of the Earth (here, the errors of his method seem to balance each other) and second, the near equivalence of the length of *stadion* used by both scholars – at least, in data sets which have been considered so far. It may not be obvious at first sight, but the recalculation of the positions given by Ptolemy in his *Geography* for the Eratosthenian size of the Earth does not involve the use of any metrical value of *stadion*. Geographical coordinates are per se dimensionless, and mathematically the procedure can be reduced to the recalculation of spherical coordinates given on a sphere with the circumference of 180,000 units to a sphere with the circumference of 252,000 units.

The simple mathematical analysis performed in this paper allows to explain many topological features of Ptolemy's world map. For example, the excessive distortion of his map is a natural consequence of the erroneously adopted size of the Earth in combination with Ptolemy's attempt to preserve the latitudes of some locations gained through astronomical observations. The other consequences are the impossibility to determine the position of Ptolemy's Prime Meridian in the geographical coordinate system *globally*, mutual displacement of the local "maps" and e.g. far too northern (or far too southern) positioning of the localitions lying approximately on the same meridian as the reference point of mapping. The erroneous

[58]The position of the Greenwich Meridian was defined through identification of the longitude of the Stone Tower with the longitude of the Daroot-Korgon. The coordinates were obtained with the formulae of the Case 2, that is, the direction and the distance towards *Sēra* was transmitted towards the position of the Daroot-Korgon before recalculation to a sphere with the circumference of 252,000 units.

[59]See Tupikova, I., Schemmel, M., Geus, K., "Travelling Along the Silk Road: A New Interpretation of Ptolemy's Coordinates", in print.

position of the *Insulae Fortunatae* can also easily be explained in the context of this mathematical approach.

It also can be shown, from a mathematical perspective, that the second scenario is fully equivalent to the first scenario Let us assume that Ptolemy was unaware of the metrical value of the *stadion* used by Eratosthenes and considered it as being equal to his contemporary unit (i.e., to 1/8 Roman mile). Then, a distance, e.g., 700 "Eratosthenian" *stadia* which Eratosthenes would have recalculated into degrees as 1°, will attain on the Ptolemaic map a value of 700 : 500 = 1.4°. In the same way, every distance dE expressed in "Eratosthenian" *stadia* would be recalculated by Ptolemy in angular measure as dP = dE * 700/500 = 1.4 dE. This is the same situation as in the first scenario: the Ptolemaic map will expand in every direction with the same multiplication factor. This case can even be reformulated in terms of an erroneously adopted size of the Earth in the following way. Assuming that the distance dE = 700 *stadia* was measured in "his" units, Ptolemy would compute "his" 1° as 700 : 1.4, that is, equivalent to 500 (real) "Eratosthenian" *stadia*. With these 500 "Eratosthenian" *stadia* per degree, his circumference of the Earth would be 360 * 500 = 180,000 "Eratosthenian" *stadia*. Thus, both cases are, from a purely mathematical standpoint, equivalent and can be treated with the same formulae. The decision between two mathematically but not historically equivalent scenarios can only be made on the basis of additional information.

It is also easy to show that an erroneous recalculation of distances transmitted in "alien" units (*parasangs*, *schoinoi*, etc.) into *stadia* for cartographical purposes can also be interpreted as the erroneously determined size of the Earth from a mathematical standpoint and can be treated with the same formulae.

The impressive precision of Ptolemaic coordinates after recalculation seems to rule out the possibility of the third scenario. Nevertheless, for comparison, we also have recalculated the Ptolemaic positions under the assumption that the length of the "Ptolemaic" *stadion* attains 185 m. Our results clearly show that such a length does not eliminate the longitudinal extension of the Ptolemaic map.[60]

One should not underestimate the huge amount of work performed by Ptolemy in collecting and evaluating geographical data. At the latest stage of his work, while creating the famous table of his coordinates, Ptolemy must have ascribed to the metrical value of a stadium some angular value obtained by measuring of the circumference of the Earth. The mistake I believe to have shown him to have made here had far-reaching mathematical and historical consequences.

Acknowledgement

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[60] A visual comparison of the recalculation for the length of the Ptolemaic *stadion* in both cases is given in Tupikova, I., Schemmel, M., Geus, K., "Travelling Along the Silk Road: A New Interpretation of Ptolemy's Coordinates", Appendix 1, in print.

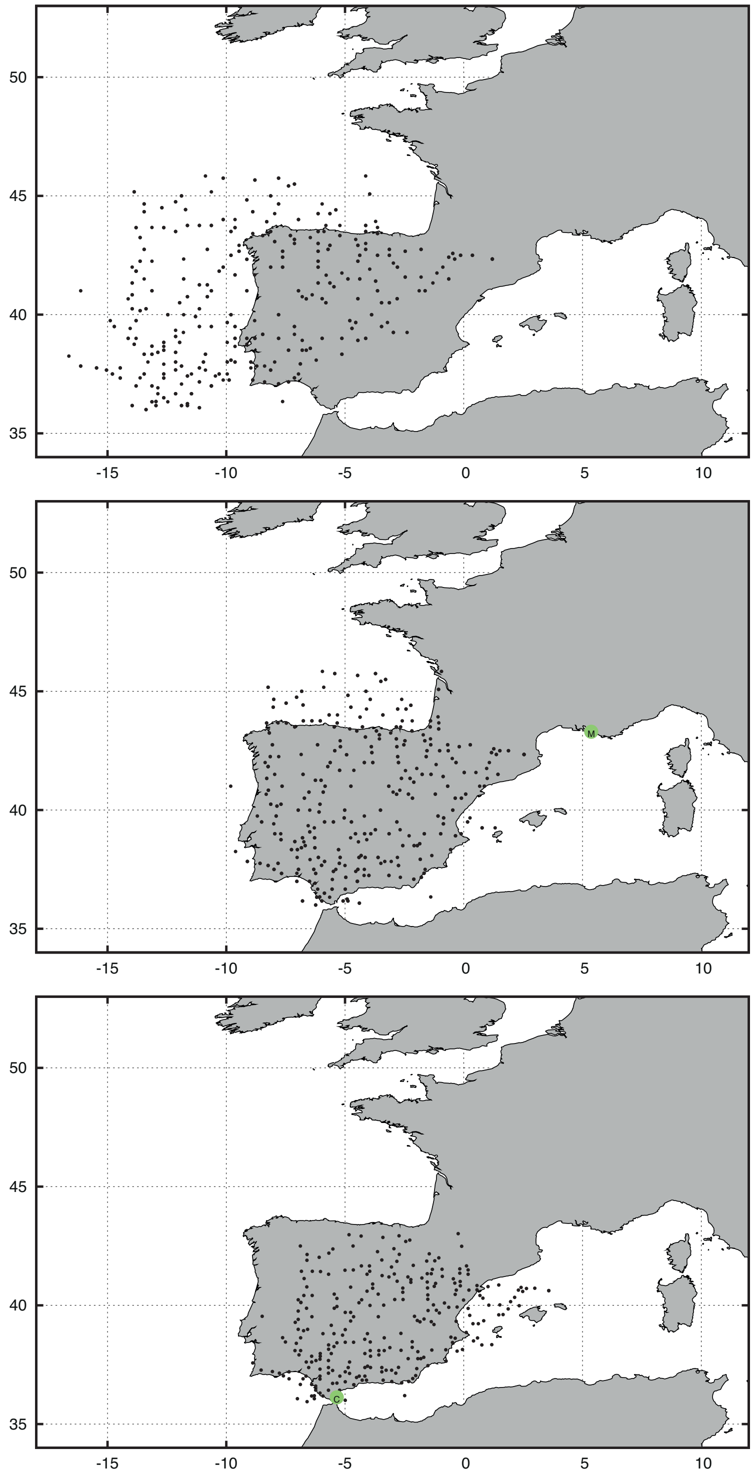

Figure 11: Identified Ptolemaic positions in Spain. Top: original Ptolemaic positions referenced to the Greenwich Meridian as defined with respect to the position of Marseille. Middle: positions recalculated relative to Marseille. Bottom: positions recalculated relative to the *Pillars of Heracles*/Gibraltar.

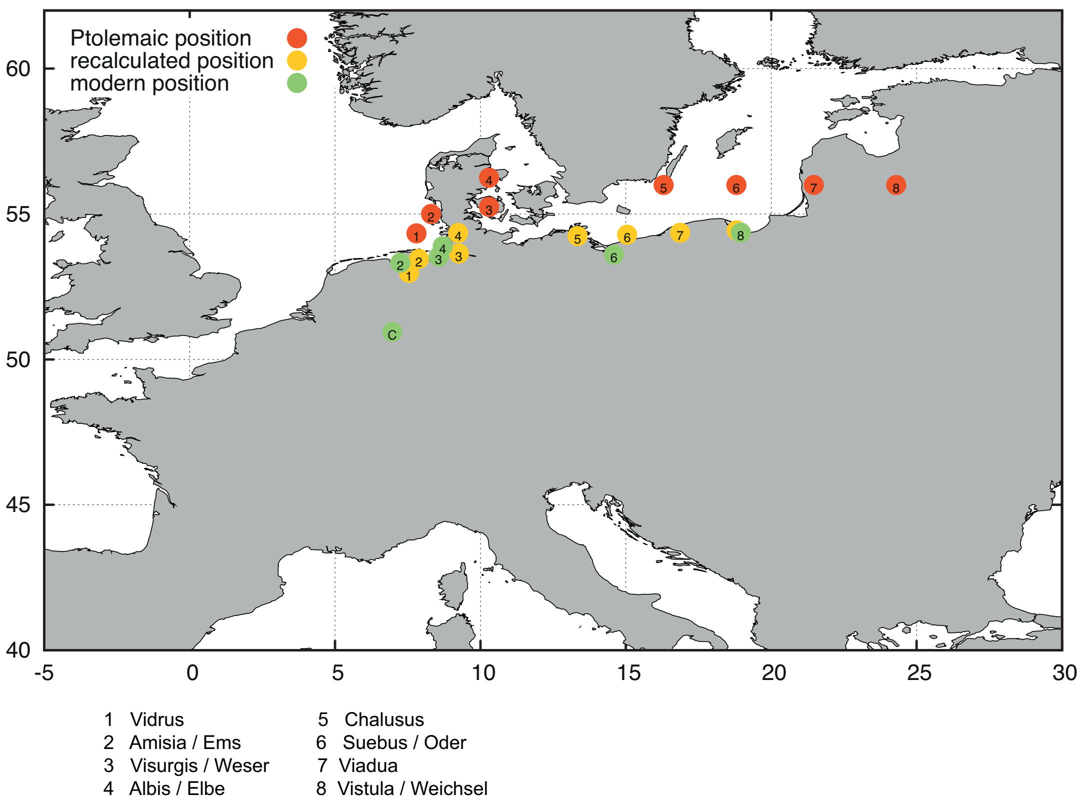

Figure 12: Recalculation of positions of rivermouths in *Germania Magna* to an Earth circumference of 252, 000 units 252, 000 units. The position of the Greenwich Meridian is defined relative to Cologne as the reference point.

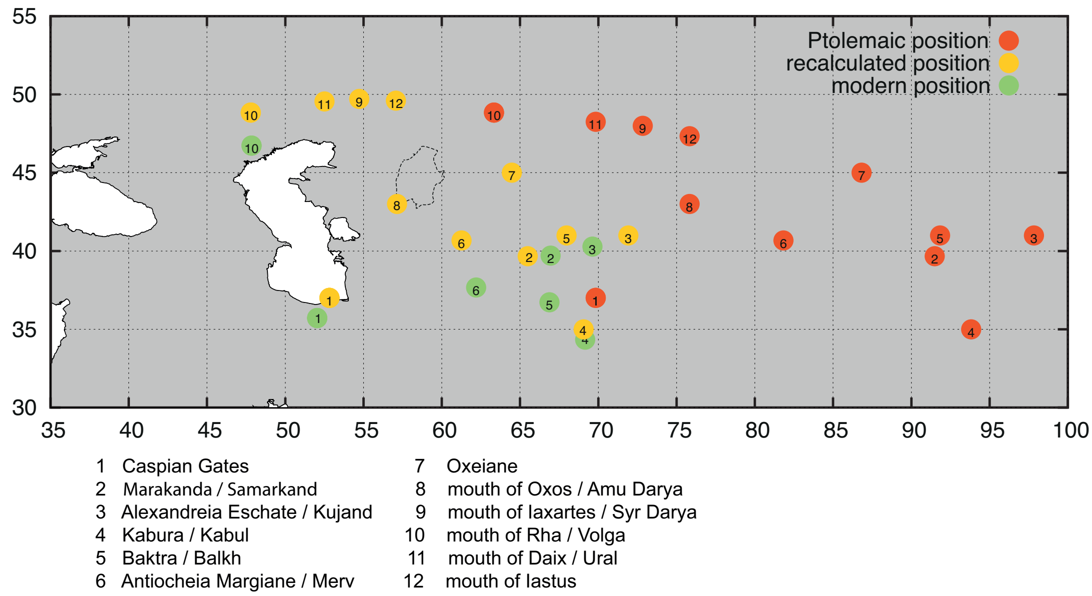

Figure 13: Recalculation of some positions in Bactria and Sogdiana to an Earth circumference of 252, 000 units 252, 000 units. The position of the Greenwich Meridian is defined relative to Rome as the reference point.

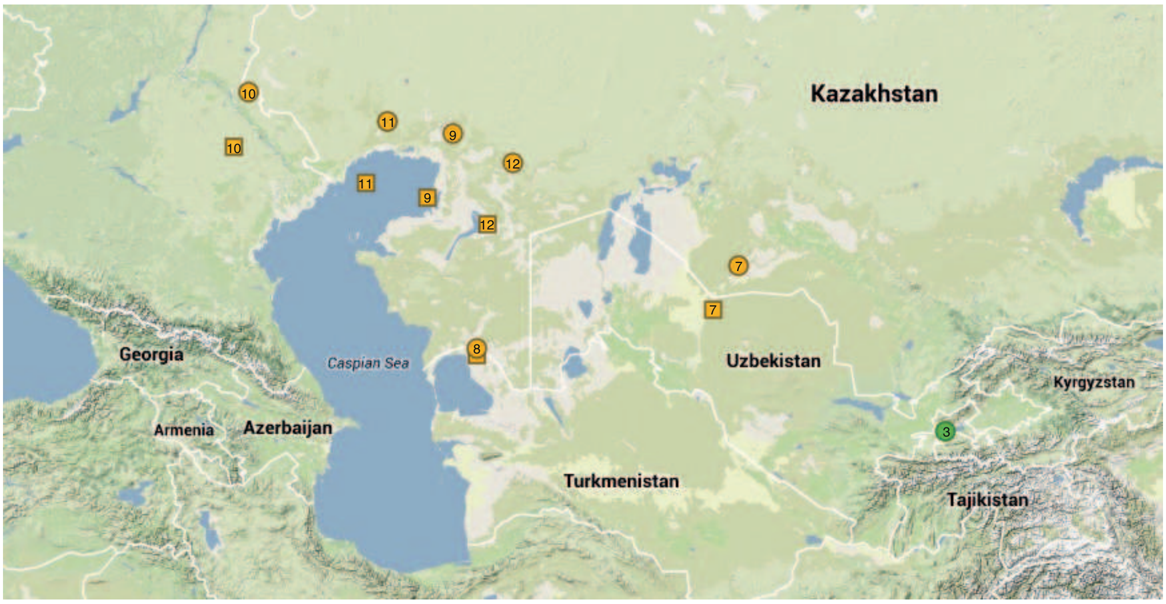

Figure 14: Recalculation of positions of the mouths of rivers to an Earth with a circumference of 252, 000 units 252, 000 units relative to Alexandreia Eschate /Kujand. Solutions obtained with the formulae of the Case 1 are marked with yellow circles and with formulae of the Case 2 with yellow squares.
Map data ©2015 AutoNavi, Basarsoft, Google, Mapa GISrael, ORION-ME, ZENRIN.

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