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Archimedes and Ship Design

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Abstract

In his written work Archimedes' primary contributions to ship design were the Law of Buoyancy and the Criterion of Stability of a floating object. These laws form the foundations of the floatability and upright stability of ships. How did he create and justify this fundamental knowledge? Might he have applied it to contemporary issues in the ship design of his era? How was this knowledge passed down through the centuries and when and how was it applied to practical ship design decisions? What is the role of Archimedes' fundamental insights in today's ship design? This article will address these issues in its three sections on Archimedes' own contributions, the history of his heritage in the maritime field and the continuing significance of his physical laws in modern ship design.

Motto on Archimedes:
All praise him, few read him, all admire him, few understand him. (A. Tacquet, 1612-1660).

1. Introduction

Archimedes by many is regarded as the most eminent mathematician, mechanicist and engineer in antiquity. He is also famous for his practical application of scientific knowledge in engineering design. His knowledge has influenced ship design for many centuries, either directly or indirectly. Yet it took two millennia before his basic insights were applied quantitatively in practice at the design stage of ships. Why this long delay? It appears worthwhile to trace the history of this tedious knowledge transfer from antiquity to modernity. How were these elements of knowledge created and justified by Archimedes, how were they passed on as his heritage and how and when were they applied in ship design practice to this day?

Ship safety and stability considerations play a dominant role in the ship design decision process. One needs fundamental knowledge, based on physical principles, to design a safe ship. This kind of knowledge may be applied in two forms:

- As intuitive, qualitative knowledge, corroborated by observation and experience, augmented by a rational understanding of the mechanisms of stability. This sort of cause and effect feeling guides in numerous tradeoff decisions on practical consequences of design measures.
- As a quantitative knowledge, based on calculations, to predict the stability performance of a new design.

Archimedes' principles have been exerting a strong influence in both categories, at least among those properly initiated. But it was a long and arduous road from his first creation and brilliant justification of the basic concepts of hydrostatics, which lay buried in rare copies of scrolls in scientific libraries in antiquity, to their broader spreading after rediscovery during the Middle Ages and their wider circulation and gradual acceptance by printed media and at last to their practical application in design calculations. They have turned into routine design tools today. It is almost a miracle that we benefit today for the safety of our maritime designs

[1] This article was written for and presented at the International Conference on "Archimedes in the 21st Century", held at New York University, NYC on the 31st of May/ 1st of June, 2013, and organized by Prof. Chris Rorres.

from ideas conceived more than 2000 years ago, then almost forgotten, but luckily resurrected and brought to useful application only after about 1750.

To recount this history and to understand how this happened the following questions will be addressed in this article:

- How did Archimedes create the fundamental knowledge and justify the laws of hydrostatics which until today form the foundation for judging ship floatability and stability in ship design?
- How might Archimedes himself have applied these insights to contemporary practical projects in ship design in his era?
- On what circuitous routes did Archimedes' insights arrive in modernity?
- How and when were the laws of Archimedes, supplemented by other fundamental methods, apt to be applied quantitatively and numerically to practical ship design calculations?
- What role do Archimedes' basic laws play today in modern ship design methodology?

It is the purpose of this paper to trace the history of the knowledge in ship floatability, stability and design created by Archimedes and passed down through many centuries to its current relevance in modern ship design.

2. Archimedes

2.1 Precursors

Human knowledge on the risks of seafaring is ancient. The first human experiences in oceangoing navigation by waterborne vehicles date back well into prehistoric times. There is indirect, though conclusive evidence of the first human settlers on the continent of then contiguous Australia/New Guinea arriving from mainland Southeast Asia between about 30000 to 40000 years ago by crossing a deep ocean water gap of about 100 miles using watercraft capable of carrying humans, animals and cargo (Diamond [1]). In the Mediterranean Sea traces of waterborne navigation date back to about 10000 B.C., notably in Egypt, Babylonia, later in Phoenicia, Crete, Cyprus and Greece. Early ocean voyages, some of them across considerable distances, required seaworthy ships, safe against all hazards of the sea. Details are documented in the literature (Kemp [2], Johnson, Nurminen [3]).

Thus at the time of Archimedes during the 3rd c. B.C. Greek shipbuilding and ship design, as he knew it, had already reached an advanced level of construction technology and design complexity for ships to be used in trade, cargo transport and even warfare. Safer and ever larger ships had evolved during many centuries. The safety of ships was judged, as today, by their ability to survive the risks of sinking and capsizing, i.e., by their floatability and upright stability. These properties had to be assessed largely intuitively on the basis of experience, observation and comparison with similar designs. It was difficult to predict the performance of new designs prior to building them for lack of physical insights as well as analytical and numerical methods of design evaluation. The rational foundations for design decisions and for safe ship operations had not yet been laid.

It was Archimedes, the eminent mathematician, mechanicist and engineer, who in a stroke of genius was able to combine these various viewpoints and to develop a rational theory of hydrostatics of floating objects, which was directly applicable to the issues of ship floatability and stability. Thus the physical principles of hydrostatics were then well understood although the practical application to ship design was long delayed, actually by about two millennia, before numerical calculations of these important elements of ship safety could be performed routinely by numerical methods at the design stage. We will discuss the reasons for this long delay in a later section. Nevertheless Archimedes and his contemporaries on the basis of his hydrostatic principles were able to judge at least qualitatively the deeper physical processes involved in ship hydrostatics and thereby to judge the potential effectiveness of certain design measures.

On the mathematical side and in the logic of rigorous proofs Archimedes had several predecessors, too, in the tradition of earlier Greek philosophy and mathematics. Table 1 gives an overview of some important precursors and contemporaries who influenced his work. It is important to single out Eudoxos [4], a pupil of Plato, who not only established rules and set standards for the rigor demanded in Greek proofs, but also worked out the Method of Exhaustion, an approximation scheme for evaluating integration hypotheses for curves and surfaces by successive refinement of inscribed or circumscribed polygons (or polyhedra). Archimedes made much use of this approximation technique, which was his tool for area, volume and centroid evaluation of simple geometric figures, while integral calculus was not yet conceived in antiquity. Archimedes was well trained in the contemporary methods of Greek mathematics, both by his early education received in Syracuse, reportedly also by his father, an astronomer, and by his contacts with and probable visit to Alexandria, an ancient center of excellence in mathematics in this era. Details on the knowledge thus available to him are found in Heath [5].

Table 1: Chronology of Precursors and Contemporaries of Archimedes

Thales of Milet (624-544 B.C.)
Pythagoras (580-496 B.C.)
Demokritos (ca. 460-ca. 360 B.C.)
Plato (427-347 B.C.)
Eudoxos (410-356 B.C.)
Aristotle (384-322 B.C.)
Alexandria founded: 332 B.C.
Euklid (325 B.C.-ca. 265 B.C.?)
Mouseion in Alexandria: 286- 47 B.C.
Archimedes (ca. 287-212 B.C.)
Eratosthenes (284-204 B.C.)

Thus Archimedes was able to build his deductions on a solid tradition in Greek mathematics and logical rigor, but he also was very creative in developing his own mathematical tools when he needed new, original ones.

2.2 On Floating Bodies

- **Preparations**

Archimedes of Syracuse laid the foundations for the hydrostatics of floating objects in his famous treatise “On Floating Bodies (OFB)” (περὶ ὀχουμένων) [6]. In this treatise he was the first single-handedly to establish the laws of equilibrium for a body at rest in a fluid, floating on top or submerged or even grounded, on a scientific basis by deduction from a few axioms or first principles. Thus although he never wrote about applications to ships he did develop the physical foundation for judging the force and moment equilibrium of floating objects, including ships, i.e., their floatability (force equilibrium) and stability (moment equilibrium).

The Principle of Archimedes, based on force equilibrium of buoyancy and gravity forces, holds for objects of any shape. The criterion of stability was first pronounced by Archimedes for the special case of homogeneous solids of simple shape, a semi-sphere and a paraboloid of revolution. These results form the cornerstones of ship hydrostatics to this day.

We are fortunate that many, though not all, of Archimedes' treatises have survived from antiquity to this day, essentially all in copies, some in Latin translation, some in Arabic, and a few even in the Greek language of the original. Many were originally lost in late antiquity and were often luckily rediscovered much later, which will be addressed in a later section. But from those which are preserved we are able to reconstruct the train of thought that Archimedes took to arrive from the principles of geometry and engineering mechanics at his scientific foundation of hydrostatics.

Table 2 gives an overview of the essential preserved treatises by Archimedes. We cannot precisely date the first appearance of these works, but there is sufficient evidence in their contents to suggest their sequence of publication. Compilations of Archimedes' works exist in several classical and modern languages (Heath [5], Heiberg [6,7], Dijksterhuis [8], Van Eecke [9], Czwallina-Allenstein [10, 11] etc.). They are essentially in agreement on the chronology of appearance. Thus it is in essence undisputed that OFB was preceded by a few other fundamental treatises, which we will briefly address here.

Table 2: Chronology of Archimedes' Preserved Treatises

Item	Title	Probable Sequence
1	On the Sphere and Cylinder, Books I and II	(5)
2	Measurement of a Circle	(9)
3	On Conoids and Spheroids	(7)
4	On Spirals	(6)
5	On the Equilibrium of Planes, Books I and II	(1) and (3)
6	The Sandreckoner	(10)
7	Quadature of the Parabola	(2)
8	On Floating Bodies, Books I and II	(8)
9	Stomachion	
10	The Method of Mechanical Theorems	(4)
11	Book of Lemmas	
12	The Cattle Problem	

- **The Law of the Lever**

In his treatise "On the Equilibrium of Planes", Books I and II, Archimedes concerns himself with the "moment" equilibrium of objects on a lever system like a balance (Fig. 1). The objects might be homogeneous solids or elements of thin planar areas of constant thickness and homogeneous gravity distribution, which can be regarded as solids so that the same lever laws can be applied in dealing with their equilibrium and centroids.

The principle of the lever and especially of the balance was certainly known since prehistoric times, e.g., in ancient Babylonia, Egypt and China (see Sprague de Camp [12], Renn and Schemmel [13]). Archimedes presumably was the first to pronounce the physical law of the lever and to apply it to many mechanical and geometric systems. The Greeks did not know the concept and terminology of moments, so he spoke of "The Law of the Lever" for the same purpose.

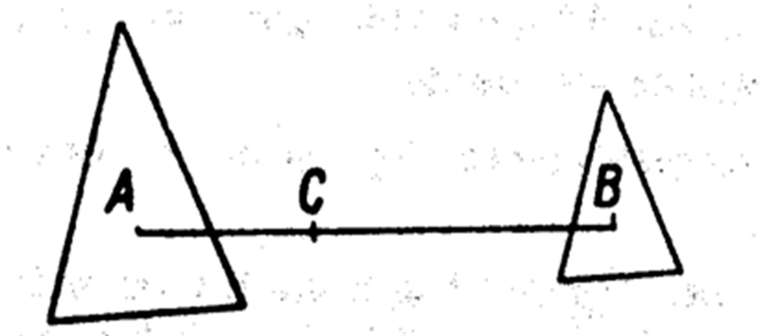

Fig.1: Lever System, Balance with Unequal Arms (from [10])

In this treatise Archimedes derives the following conclusions from the Law of the Lever:

- The equilibrium of unequal weights on a balance of unequal arms (Fig. 1) so that the weights are inversely proportional to their lever arms ("moment equilibrium").
- How to lump two or more objects into a single compound object with a compound centroid so that the statical moment of the compound object remains the same as for the sum of the separate objects.
- Removing, adding or shifting objects in a system and finding the new centroid.

To summarize, in this treatise Archimedes pronounces the following principles, to which he can later resort for the stability of floating bodies:

- The Law of the Lever (moment equilibrium).
- Lumping objects into their centroids, thus forming resultants.
- Finding compound centroids for a set of system components.
- Removing, adding or shifting objects in a system and its effect on the centroid.

- **The Method of Exhaustion**

In evaluating the area of planar figures or the volume of geometric solids the ancient Greeks met with two major obstacles:

- They had no concept of real numbers, let alone irrational or transcendental numbers, nor did they have a consistent system of units for measuring length, area and volume. They circumnavigated this difficulty by restricting themselves to finding only the ratio of a figure area or volume to that of a known figure, e.g., a square or a triangle whose size is known. In this context real numbers were expressed by ratios of integers. Irrational or transcendental numbers were approximated as such ratios. E.g., finding the area of a circle relative to a circumscribed square led to the famous "quadrature of the circle" type of problem.

- They had no method to evaluate areas and volumes of figures equivalent to modern calculus. This is why they resorted to approximation methods based on a finite number of successive subdivisions of the given figure by simple shapes whose area or volume are known. The Method of Exhaustion, as it was named much later in the 17th c., is such an approximation method based on successive refinement of the result by means of a polygonal approximant whose deviation from a given figure shall be made as small as desired after a sufficient, finite number of subdivision steps. The method is not equivalent to integral calculus since a limiting process to infinitesimal step size is not performed. But for geometrically well defined figures of simple shapes very accurate approximations can be obtained after a finite number of steps.

To illustrate how Archimedes used this method let us refer to to his treatise “Quadrature of the Parabola”, §21-24, where he derives the area of a parabola segment AZBHC, compared to an inscribed triangle ABC whose apex is B such that the center line BD halves the parabola segment.

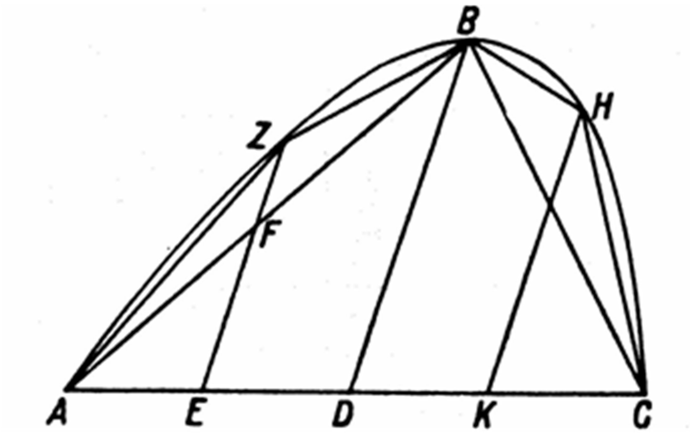

Fig. 2: Parabola Segment AZBHC and Inscribed Triangles (from [10])

The Method of Exhaustion then proceeds as follows:

Two additional triangles AZB and BHC are constructed between the parabola and the original triangle by drawing parallels to the center line BD through the quarter points E and K of the base line. For the parabola it can be shown that each of the new triangles has an area of (1/8) of triangle ABC, hence both together of (1/4) of the original triangle.

In a process of successive refinement in each step new triangles are added between the polygon sides and the parabolic arc, always halving the intervals along the base line. It can be shown that for the parabola each new set of triangles adds (1/4) of the area of the preceding set of triangles. Thus in a process of continuing refinement the terms of triangle areas are added to a geometric progression whose quotient is (1/4). The sum of this progression is either known on arithmetical grounds, viz., for the parabola, or estimated after a finite number of steps when the truncation error appears small enough. For the parabola the end result is (4/3) times the area of the first triangle ABC. Such a result is first asserted inductively, then proven rigorously by reductio ad absurdum of any deviating results. Details of this proof can be found in Archimedes’ original treatise (cf., e.g., Heath [5] or Nowacki [14]).

Archimedes, who seems to have adopted this method from Eudoxos, frequently used it for the evaluation of areas and volumes of simple shapes, i.e., for certain applications where modern analysis would use integral calculus. This explains why certain of his results were limited to simple geometric shapes, e.g., for the hydrostatic stability of floating objects, and were not extended to objects of arbitrary shapes, e.g., ships.

- **<u>The Method of Mechanical Theorems</u>**

In certain cases Archimedes also had another fast and efficient method available to derive hypotheses for geometric results from mechanical analogies. This technique is described in his treatise "The Method of Mechanical Theorems", which was long lost, but then was rediscovered first by J.L. Heiberg in 1906 in a Greek monastery, the Metochion, in Constantinople in an old 12th c. palimpsest. The Archimedes text had been rinsed off and a Greek prayerbook (euchologion) was written on the same vellum sheets. But Heiberg was able to decipher most of the original Archimedes text under a magnifying glass, to document, transcribe and translate it into German [15]. The interesting history of this palimpsest is described in more detail in Sections 3.2 and 3.3.

In this treatise Archimedes explains to Eratosthenes how he applies principles of mechanics in geometric reasoning to obtain inductive conclusions on geometric facts. Mechanical theorems are based on observation, hence inductively founded. Archimedes therefore uses these methods only to help propose hypotheses on geometrical facts. Archimedes does not regard these results as validly proven, but a strict, deductive, purely geometric proof must follow to confirm the conjecture. Some of these subsequent geometric proofs are preserved, others are lost.

In his treatise "The Method…" Archimedes deals with the example of a solid paraboloid of revolution. He asserts:

"The centroid of a right-handed conoid (here a paraboloid of revolution) cut off by a plane at right angles to its axis lies on the straight line which is the axis of the segment, and divides the said straight line in such a way that the portion of it adjacent to the vertex is double of the remaining portion".

The proof, based on the Law of the Lever and "The Method…", is presented in detail in the treatise "The Method…" (Heath [5]), proposition 5, and is also fully explained by Nowacki [14]. This confirms that the centroid of the paraboloid segment is located at 1/3 of segment height above the base line or 2/3 below the summit. Thus Archimedes was able to demonstrate geometric facts without needing to resort to calculus. Archimedes later uses this result in the context of his study on hydrostatic stability of a paraboloid in OFB.

- **<u>The Principle of Archimedes</u>**

The fundamental law of hydrostatics for a body at rest within or on top of a homogeneous liquid is pronounced and justified by Archimedes in his treatise OFB, Book I. It is deduced here strictly by an experiment of thought, i.e., without any experimental observation or other empirical basis. It holds for a body of arbitrary shape.

The liquid is assumed to be homogenous and such that any liquid particle is pressed downward vertically by all particles in the vertical line above it (OFB, Book I, § 1). We would call this a hydrostatic pressure distribution in modern terminology, but the Greeks did not know the concept of "pressure" in antiquity. In Book 1, §5 Archimedes asserts for a body in a liquid at rest:

"A body submerges in a specifically heavier liquid to the extent that the volume of the liquid displaced by it weighs as much as the whole body".

In modern terminology this <u>Principle of Archimedes</u> can be stated as:

$\Delta = \gamma\ V$, where
Δ = displacement = weight of body
V = displaced volume
γ = specific weight of liquid
$\gamma\ V$ = weight of displaced liquid volume =
= buoyancy force

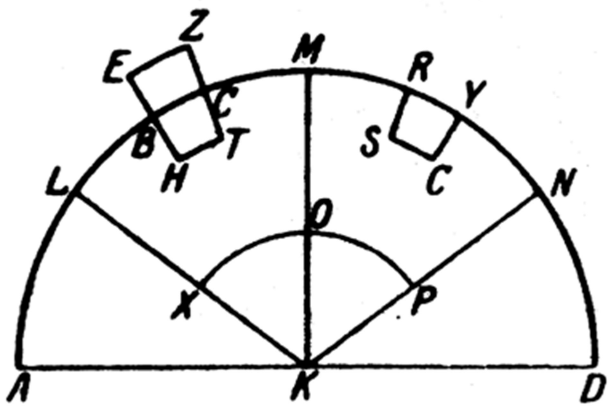

Fig. 3: Proof of Archimedes' Principle (from [10])

Archimedes' proof in OFB, Book 1, §5 is brilliantly elegant and brief (Fig. 3):

- The surface of any liquid at rest is a spherical surface whose center point is the center of the earth (section ALMND).
- The body EZTH be specifically lighter than the liquid, hence it floats in the surface.
- We consider two neighboring equal sectors of the sphere, bounded by the surfaces LM and MN. The first sector contains the floating body whose submerged part is BHTC. The second sector instead has an equal volume RYCS filled with the liquid.
- The liquid is at rest. Thus the surfaces between XO and OP experience identical pressing loads. Thus the weights of the volumes on top of these surfaces are equal. But the weight of the liquid in the first sector, aside of the space BHTC, equals the liquid weight in the second sector, aside of the space RYCS. Therefore clearly the weight of the body EZTH equals the weight of the liquid volume RYCS. It follows that the liquid volume displaced by the body weighs as much as the whole body.

Note that this concise proof of the Principle of Archimedes is strictly deductive and is based entirely on an experiment of thought and very few axiomatic premises. It holds for arbitrary body shapes in an arbitrary type of liquid. It was derived for liquids and bodies at rest without knowledge of the pressure distribution anywhere.

- **The Eureka Legend**

The famous Eureka legend about Archimedes' discovery of a hydrostatic law in the bathtub, which goes back to Vitruvius ([16], Book IX.3, published around the birth of Christ) is often misinterpreted and has led to confusion regarding this discovery. According to Vitruvius Archimedes was challenged by King Hieron to determine whether a wreath, made for the king by a goldsmith for a votive offering to the gods, was of pure gold or fraudulently made of gold mixed with silver. Archimedes is said to have observed how the water displaced by his body in a brimful bathtub was a measure of the displaced volume, which he regarded as a breakthrough in solving the king's wreath problem. Archimedes was apparently so elated by this discovery that he jumped up from the bathtub and ran naked through the streets of Syracuse shouting "Eureka" ("I have found it")! Vitruvius goes on to report how Archimedes then demonstrated the fraud. The wreath and two equally heavy pieces of pure gold and silver successively were each sunk in a bowl full to the brim of liquid . After removing the object its volume was determined by refilling the bowl to the brim and measuring the weight of the replacement liquid. This indeed gives a clue as to the relative densities of the two metals and the wreath, viz., the weight of the object related to the weight of the displaced water.

Thus Archimedes had discovered a method for measuring the volume of a submerged object and thus, if the weight of the object is known, its specific weight (weight per unit volume). This is sufficient to prove the fraud of using a lighter, false metal in the wreath. The specific weight of the objects is used as a criterion of comparison.

But the Principle of Archimedes cannot be proven by a human sitting in the bathtub. The human body in this scenario is usually supported in part by a ground force so that the equality of body weight and buoyancy force does not hold. Thus we must credit Archimedes for deducing his famous Principle strictly by experiment of thought, as explained. Is it not another sign of his brilliance that he was able to deduce this Principle without resorting to a physical experiment, let alone an observation in a bathtub?

The legend of the wreath has attracted many critical reviews debating the practical difficulties in realizing sufficiently accurate results. Recent reviews and experimental reconstructions of the wreath experiment are given by Costanti [17] and by Hidetaka [18]. There seems to be agreement among these scholars that it is feasible to reconstruct the experiments described by Vitruvius in a small enough bowl, though not in a large bathtub, in order to solve King Hieron's fraud problem and to measure specific weights of solids, though not in order to find the Principle of Archimedes. The bathtub, if anything, might have served as an inspiration.

- **Hydrostatic Stability of Floating Objects**

In his treatise OFB Archimedes also deals with the stability of bodies floating on the surface of a liquid, especially of homogeneous solids of simple shape. His basic ideas are already shown in Book I, §§8-9 for a segment of a sphere. In Book II, §§ 2 ff. the example of an axisymmetric paraboloid segment as shown in Fig. 4 is even more illustrative and will be discussed here.

The stability criterion for hydrostatic equilibrium is based on the experiment of thought to incline the body from its upright condition and to determine whether the resultant gravity and buoyancy forces acting on the body in this condition will tend to restore it to its upright condition. The angle of inclination (heeling angle) is finite, but so that the base of the paraboloid segment does not get wetted. For the stability of this body Archimedes asserts in Proposition II.2:

"A homogeneous solid paraboloid segment cut off perpendicularly to its diameter, whose axis is not greater than 1.5 times the paraboloid's halfparameter, whatever its specific weight, if it floats in a liquid so that its base does not touch the liquid surface, will not remain at rest unless its axis is vertically oriented, but will restore itself to the upright condition".

The proof is based on finding the centroid R of the homogeneous solid, through which the weight resultant D1+D2 is acting downward, i.e., the center of gravity in our terminology, and the centroid B of the submerged volume, i.e., the center of buoyancy, through which the the equal and opposite resultant buoyancy force B1+B2 is acting upward. The lever arm between these two forces must be of such orientation that a positive restoring tendency ("moment") results, which corresponds to our familiar positive righting arm requirement. However there is a subtle difference in Archimedes' demonstration for the homogeneous solid: For the submerged part of the inclined solid alone its center of gravity and its center of buoyancy are in the same position B so that this submerged part produces no lever arms (unlike in a ship). Thus it is sufficient to show that the gravity force D2 of the above surface part of the solid and its equal, but opposite counterpart, the incremental buoyancy force B2, which acts through the center of buoyancy B, have a positive restoring arm. One can show that, if this "incremental righting arm"of B2 and D2 is positive, then the conventional righting arm of the forces (B1+B2) and (D1+D2) is also positive.

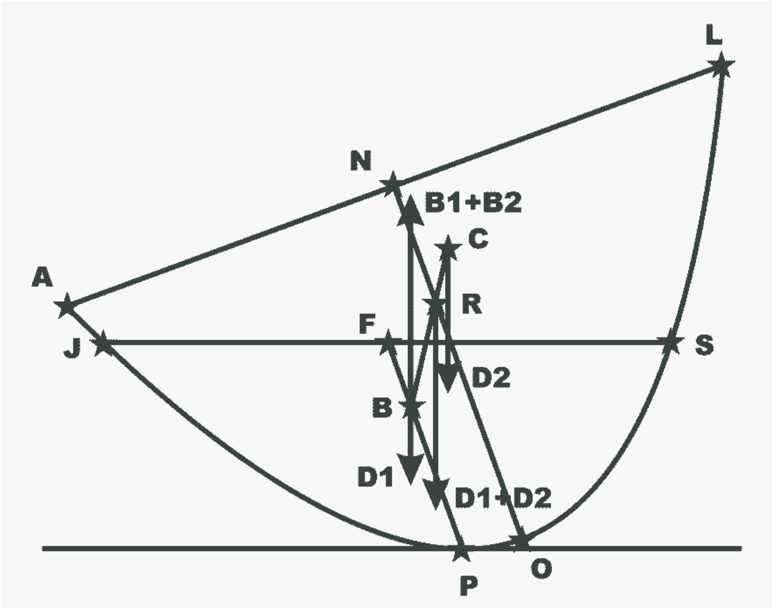

Fig. 4: Inclined Paraboloid Segment (from [14])

In the actual proof Archimedes applies several mechanical and geometric principles which he deduced in this treatise or in earlier work. For more details see also Nowacki [14]. He proceeds in the following steps:

- The paraboloid is intersected by a vertical plane in the parabola APOL and is inclined by a finite angle. The horizontal plane of the liquid surface intersects the paraboloid in JS. A tangent parallel to JS touches the parabola in P.
- A parallel line to the axis NO, drawn through P bisects the chord length JS at midpoint F (proven in earlier in "Quadrature of the Parabola", §19). PF is thus the axis of the submerged part of the paraboloid.
- Archimedes now uses his theorem on the centroid of the paraboloid segment lying on its axis at a point 2/3 of the axis length above the summit, proven in his treatise on "The Method…". Therefore PB = (2/3) PF. Thus B is both the center of buoyancy and the center of gravity of the immersed part. The immersed part therefore creates no moments and can be left out of consideration for moment equilibrium. Further let OR = (2/3) ON, so that R is the center of gravity of the whole solid.
- Using the centroid shift theorem from his treatise "On the equilibrium of planes", Book 1, §8, Archimedes constructs the center of gravity C of the above surface part JALS: If the immersed part JPOS, centroid B, is removed from the whole solid, centroid R, then the remaining above surface part JALS has a centroid C on the straight line BR, extended beyond R, such that

RC:RB = (immersed volume) : (above surface volume)

- Thus C lies on one side, B on the other side of R.
- Therefore the vertical gravity force D2 through C and the equal, but opposite buoyancy force B2 through B are not acting in the same vertical line, hence not in equilibrium of moments, but will tend to restore the paraboloid to its upright condition. Thus the investigated paraboloid is hydrostatically stable.

Although this derivation holds only for the homogeneous solid paraboloid, it can be shown that a similar reasoning can be developed for a solid of any shape and with non-homogeneous mass distribution, hence also for ships. The incremental righting arm of the paraboloid is thus the ancestor of the conventional righting arm used in modern stability analysis. Positive righting arms are a necessary condition of upright ship stability.

- **Achievements and Deficits**

To summarize the state of knowledge in the hydrostatics of floating bodies achieved by Archimedes, as displayed in his treatise OFB, and to state the elements of information still missing for a complete analytical and numerical evaluation of the hydrostatic properties of a ship design, the following presents a synopsis of his achievements and remaining deficits.

Archimedes contributed the following fundamental insights and methods pertinent to the hydrostatics of ships:

- Archimedes defined the resultant gravity and buoyancy forces (displacement and buoyancy) acting on a floating body and pronounced the force equilibrium principle of their equality in the same line of action and in opposite directions (Principle of Archimedes).
- He devised methods for calculating the resultants of buoyancy and weight acting through the CB and CG, at least for solids of simple shape (Method of exhaustion, Method of Mechanical Theorems, Lumping of System Components into Compound Systems).
- From the axiom of moment equilibrium Archimedes deduced a criterion for hydrostatic stability by introducing the concept of righting moments or righting arms based on the couple of buoyancy and displacement forces.

With this information a qualitative understanding of which design measures have favorable or unfavorable influence on the design was at least possible.

To perform a complete, quantitative analysis of the hydrostatic properties of a ship at the design stage the following elements were still missing for Archimedes:

- A complete, continuous hull form definition for arbitrary ship shapes in whatever medium (mold, model, drawing).
- A method to calculate the volume and volume centroid of the underwater hull form (center of buoyancy, CB) for an arbitrary ship shape.
- A practical scheme to calculate the weight of all component parts of the ship and therefrom the compound center of gravity, CG, of the entire ship.
- Data for the external heeling forces and moments acting by wind and seaway on the ship in order to assess the required safety margins in floatability and stability.

In view of these deficits Archimedes and his contemporaries were not yet able at the design stage to quantitatively predict the design's draft, trim and heel angles, and stability measures for the hull in empty and loaded conditions. This remained a matter of experience, observation and empirical judgment. In case of doubt ballast might be taken in the lower hull to improve a ship's initial stability.

- **Archimedes' Role in Practical Ship Design**

Archimedes shared with traditional Greek philosophers the Platonic aristocratic disdain of publishing any results of practical projects so that we have no evidence in his own writing of his involvement in technical inventions and engineering design. But we know from other, often much later writers (Vitruvius [16], Diodorus of Sicily, Plutarch[24] and others) that he excelled in innovative engineering work and in creative inventions for very practical purposes. His conception and realization of many practical devices and unique, original machines are well documented (e.g. by Dijksterhuis [8]). This has often raised the question to what extent Archimedes might also have been involved in practical ship design and ship construction projects.

The most prestigious shipbuilding project in Syracuse during Archimedes' lifetime was without doubt the design and building of the Syrakosia, a giant cargo ship, mainly for the transport of grain, an export commodity from Sicily. King Hieron II of Syracuse had ordered the ship to be built around 235-230 B.C. in Syracuse. We are curious to know whether Archimedes was involved in this representative project and was able to apply his theoretical knowledge in practice.

The most detailed technical report about the Syrakosia stems from Athenaeus who documented many details of the design and the history of this ship in his treatise "Deipnosophistai" [19], published somewhat after AD 192, thus more than four centuries after Archimedes and the existence of this ship. The ship impresses by its size, cargo capacity, accommodation and outfitting. A thorough reconstruction was recently performed by Bonino [20]. According to this source some of the main features of the ship were:

Length = 80.0 m	Beam = 15.5 m
Draft = 3.85 m	Depth = 5.6 m
Displacement = 3010 metric tons	Payload = 1940 metric tons
Block coefficient of submerged part of hull = 0.615	

The ship had three masts and three decks. The combat deck had 8 towers with war machines. The complement was 825 people on board. 20 horses with stables were accommodated, too. The accommodation spaces were of high, comfortable standard, with luxurious tiling on the floors, walls and ceilings. A model of the ship was built by Tumbiolo based on Bonino's reconstruction drawings. Further details on the design and the history of the ship can be found in Bonino [20].

King Hieron had recruited the prominent naval architect Archias from Korinth to take the overall responsibility for the design and building of the ship. Archimedes is mentioned by Athenaeus as a supervisor (ό γεωμέτρης ἐπόπτης, the geometer supervisor) or perhaps just advisor in this project. It is recorded that he played a decisive role in the launching of the ship. When the lower half of the hull had been assembled on the slipway with a lead sheathing already attached, she was ready to be launched in order to add the top half later when she was afloat. But she would not move down the slipway. Archimedes was called to help. He mounted a windlass with a multiple pulley system to the hull to drag her down into the water. He was able to crank the windlass alone and to launch the ship single-handedly. He was much admired for this ingenious solution. It is also conceivable that Archimedes designed some military equipment to be placed on board for the ship's self-defense, e.g., catapults to fire arrows at enemy or pirate ships or a stone-hurler to shoot a stone of 180 pounds or a javelin of 18 foot length some 600 foot distance. Thus he was certainly welcome to contribute such ideas in his area of expertise.

But it very unlikely that he was prepared to offer an explicit stability analysis for the ship.The main reasons were explained in the preceding section: The lack of a continuous hull form definition, the difficulty in calculating volumes and centroids for an object of arbitrary shape as well as the missing experience and data for setting target values for safe stability margins. Rather practitioners would know how to use ballast and cargo deep in the hold to improve stability when needed.

Recent discussions in the literature (Zevi [21], Pomey and Tchernia [22], Pomey [23], Bonino [20]) are in agreement with this cautious opinion on Archimedes' possible involvement in practical ship design.

3. The Heritage

3.1 The Commentators

Archimedes' scientific work is quite authentically, though not completely, preserved in copies of his manuscripts although some copying errors, translation flaws and lacunae cannot be ruled out. Thus it is important to know the history of the manuscript copies and to examine their quality.

In the Hellenistic era and in late antiquity most of his manuscripts were collected in the great library of scrolls in Alexandria, Egypt at the Mouseion (286 - 47 B.C.) and later at the Serapeion (through AD 391).

Archimedes most likely had spent an extended study period in Alexandria around 240 B.C., as mentioned by Diodorus of Sicily. At the Mouseion many famous scientists lived and worked there together. The Mouseion served as a center for collecting many thousands of scrolls, but also for copying and distributing this body of knowledge within the ancient scientific world. There Archimedes had met many contemporary scientists, e.g. Konon, Dositheos, Aristarchos, Eratosthenes among others, with whom he established lifelong friendships and communications. He sent them his manuscripts and thus probably made them available to the ancient scientific world via Alexandria.

Unfortunately the libraries there fell victim to great fires and lootings at least twice, the Mouseion in 47 B.C., the Serapeion later in AD 391. Yet copies of Archimedes essential works existed elsewhere in other libraries and scientific centers.

Table 3: Historians and Commentators of Archimedes

Polybios (201-120 B.C.)
Cicero at tomb in Syracuse: 75 B.C.
Livy (59 B.C.- AD 17)
Vitruvius (ca. birth of Chr.) [14]
Diodorus of Sicily (ca. birth of Chr.)
Plutarch (AD 46-120.) [24]
Pappos of Alexandria (AD 290 -?)
Serapeion library burnt: AD 391
Proclus (AD 400-ca. 485)
Eutokios of Askalon (AD 530-600)
Tzetzes, Byzantine writer: 12^{th} cent.

Table 3 shows the names of some of the main historians, biographers and commentators on Archimedes and his work during the centuries following his death. In this period access to his manuscripts still comprised a few that are now lost. This interest, mainly in the mathematical and mechanical treatises, continued for several centuries. Unfortunately OFB is nowhere mentioned by these authors.

3.2 The Manuscripts

Archimedes own manuscripts are all lost. But fortunately several copies were made in antiquity and the early Middle Ages so that a total of 12 treatises of his are preserved and currently known (Table 2). Some of these manuscripts have survived in the original Greek language, others in Latin translation, a few even in Arabic.

In Alexandria, from where many of the master copies originated, Caesar had confiscated many scrolls from the Mouseion as war booty in order to ship them to Rome, but apparently most were burnt or lost during the uprisings in 47 B.C. The majority of the remaining scrolls were lost when the Serapeion temple, which served as library, was set on fire by a Christian mob in AD 391 (Sprague de Camp [12]).

The Byzantine Empire was best positioned geographically and suited culturally to resurrect the classical Greek traditions. In fact, Leon of Thessaloniki, a Byzantine cleric, in the 9th c. undertook a collection of Archimedes' dispersed works to which we owe the existence of at least three later master copies, called Codices A, B and C by Heiberg [7] later, which became the master sources for all later preserved copies and translations. The treatise OFB was contained in Codices B and C.

In Sicily during the Norman and Hohenstaufen rule in the 11th and 12th c. a blossoming of classical literature and science occurred, promoted in part by exiled Byzantine scholars, who apparently brought at least two sets of Archimedes manuscript copies with them to the West. When the Hohenstaufen empire collapsed after the battle of Benevento in AD 1266, these copies ended up in the papal libraries.

One of these sets, Codex A, in Greek without the OFB treatise, changed hands a few times, was however copied before it was irretrievably lost in 1564. For several centuries these copies of Codex A were the only available sources for Archimedes in Greek language.

Evidently a second set of manuscripts by Archimedes had existed in Sicily and then in the papal library, which contained OFB. This made it possible to Willem van Moerbeke, a Flemish Dominican monk, who worked as a translator at the papal court in Viterbo from 1268 to 1280, to produce a Latin translation of Archimedes' preserved works based on both earlier sources and including OFB, published in 1269. This Latin translation, later called Codex B by Heiberg [7], and its copies provided the principal access to Archimedes for the Latin reading community.

After about 1500 by means of the fast spreading of Gutenberg's printing press several printed editions of Archimedes' work soon appeared in order to satisfy the growing interest in classical science. Among the first few we mention the editions by Tartaglia (Venice, 1543) and Curtius Trajanus (Venice, 1565). Both were Latin translations, based on Codex B, the former with Book I, the latter with both books of OFB. Commandinos Latin editions (Venice, 1558/1565) based on Codex B, the 1565 edition with OFB, were highly regarded for their quality. After 1600 many other editions have appeared in Greek, Latin and in modern languages (Dijksterhuis [8]).

Clagett [25] has presented a very thorough survey of the history of Archimedes' works during the Middle Ages and through the revival in the Renaissance.

3.3 Codex C

Miraculously in 1906 a third master copy of Archimedes' collected works in Greek was rediscovered by J.L. Heiberg [15], a Danish scholar of classical languages, in a Greek monastery, the Metochion in Constantinople, later called Codex C. This copy was found in a palimpsest where Archimedes' text of the 10th c. had been rinsed off and the parchment pages were reused and reassembled in the 13th c. for a Greek prayerbook (euchologion). As Heiberg personally writes in the preface of his German translation of "The Method", which appeared in 1907 [15]:

"Last summer I have examined a manuscript at the Metochion (in Constantinople) of the Monastery of the Holy Sepulchre in Jerusalem that underneath a prayer book (euchologion) of the 13th c. contains treatises by Archimedes written in the beautiful minuscula of the 10th c., which is only rinsed off, not scraped off, and is reasonably legible with a magnifying glass. The manuscript, no. 355. 4to, stemming from the monastery of St. Sabba near Jerusalem, is described by Papadopoulos Kerameus, who gives a sample of the writing below. From this it was immediately clear to me that the old text was Archimedes…… It is however more important that the manuscript contains the very nearly complete text of "On Floating Bodies", from which in the past only the Latin translation by Willem van Moerbeke was available; its numerous lacunae and grave corruptions can now be healed completely".

Heiberg's preface goes on to praise the value of the first ever discovery of "The Method…".The new findings from this palimpsest were soon transcribed, documented in Greek and used in subsequent translations [7].

The adventurous history of Codex C continues in style: During the upheavals of the Greek-Turkish war of 1920-1922 the collections of the Monastery Library in Constantinople are taken to Greece. There the palimpsest disappears. It seems to have been acquired by a French private owner. Its condition further deteriorated, also by failed attempts to sell it as a medieval prayer book with added illuminations. Incredibly enough it eventually resurfaced at an auction by Christie's in New York in 1998, where it was bought by an anonymous American buyer for 2.2 million dollars.

Since then in 1999 the new owner lent the manuscript to the Walters Art Museum in Baltimore for secure conservation and renewed scientific evaluation. The museum team has been applying the most modern techniques for reconstructing the ancient text as accurately as possible. Advanced optical and computer-based methods have been used (multi-spectral imaging and confocal microscopy a.o.) to increase the contrast for the rinsed-off text and to focus on layers beneath the parchment surface to reconstruct the text from the badly damaged palimpsest.

Important results of this painstaking work have been published by members of the Baltimore team (Netz, Noel [26], Noel, Netz, Tchernetska, Wilson [27]). From these results the excellent quality of the reconstruction is evident, and many new findings were reported that go well beyond the status reached by Heiberg, thus closing gaps in the text and removing lacunae. Thereby our understanding of OFB has been largely confirmed and improved in details. The first ever finding of "The Method" adds much substance to our insights into Archimedes' lines of thought.

3.4 The Treatisers

During the 15th to 17th centuries a tradition developed in all major European seafaring nations to document the existing and evolving shipbuilding knowledge, whether practical or more theoretical, in more or less learned treatises for diverse purposes. The authors are often called treatisers. They came from practical shipbuilding or more theoretical background or sometimes both. The treatises served as technical notebooks, as introductory texts for the general public or just as an opportunity to display scientific and technical expertise. Shipbuilding technology and design methodologies underwent major changes during this period in practical and scientific knowhow, so the treatises captured valuable elements of contemporary background knowledge. The treatises can serve to monitor the changes that occurred in ship design and production.

We will take a quick survey of the more essential treatises, mainly in order to identify traces of Archimedean knowledge during this period when Archimedes' written work began to be more widely circulated in print. But before this knowledge in ship design could be applied, a few major prerequisites had to be met: A complete, continuous hull form definition and an accurate methodology for evaluating areas, volumes and centroids were indispensable.

During the Middle Ages and Italian Renaissance Venice was a leading sea power and shipbuilding center in the Mediterranean. Some of the earliest treatises from Venice document the shipbuilding methodology practiced there. It is found in the treatises by Michael of Rhodes [28], ca.1434/1435, Trombetta [29], 1445, and later with more textual elaboration by Drachio [30], 1599. The ships were built "skeleton-first", i.e., a skeleton of transverse frames from the keel through the bottom and continued all the way up in the sides was erected first to which the hull planking was attached later. Thus the shape was predefined by the outer edges of the skeleton frames. The Venetians and other Mediterranean shipbuilders used a special "lofting" method to lay out the shape of the transverse frames by means of a "sesto", i.e., a planar template for the master cross section of the hull with several schemes of marking, from which the shape of any other section at other stations along ship length can be derived by unique rules. Thus geometrically the frames of the skeleton are uniquely deduced from the master section by a process of translation, rotation and clipping. Thus the hull surface is fully defined by the sesto (except for the ends of the ship). See e.g. Alertz [31] for more details. In these treatises no reference to design calculations and to Archimedes' OFB are found.

In France, Spain, in Genova and at other Mediterranean cities very similar lofting methods were used. In French the "maître gabarit" is the equivalent of the "sesto". The French approach, also called the Mediterranean method, is well documented by Rieth [32]. Design calculations and reference to OFB are also still absent in the pertinent treatises.

Portugal, a successful seafaring nation during the Age of Exploration, also held a strong position in shipbuilding. Early Portuguese treatisers are Oliveira [33] 1580, Lavanha [34] 1614/16 and Fernandes [35] 1616. They deal primarily with ship geometry, molding rules and ship construction. Lavanha develops precise ship drawings, Fernandes already presents a rudimentary ship lines plan. These sources however do not contain hydrostatic calculations nor references to Archimedes.

In England William Bourne ("Treasure for Travaylers" [36]), 1578, one of the first treatisers there, already describes an approximate practical method to obtain a ship's volume estimate by taking its offsets when on a dry ground using measuring rods relative to some suitable reference plane outside the hull and up to the desired waterline. The offsets are then linearly connected to estimate cross-sectional areas up to the desired draft. This is done in several transverse sectional planes along ship length. The volume of any segment between measured neighboring cross-sections at any given draft is then approximated by linear interpolants. Thus a rough volume estimate is obtained for the ship, which is converted to the ship's weight or displacement on that draft based on the Principle of Archimedes.

Other English treatisers (Mathew Baker/John Wells, 1570-1627, cf. R.Barker [37], R. Dudley [38], 1646, Bushnell [39], 1664) further evolved methods for designing and molding ship geometry leading up to first ship lines plans on paper. However Anthony Deane [40], 1670, undertook the next steps of section area planimetry from lines plans by circular arc or rectangular/triangular approximants as well as volume estimation, segment by segment, between cross-sections. The Principle of Archimedes was again used to find the displacement for any given draft. Stability analysis was not attempted.

During the same period early French treatisers (Fournier, 1643, Pardies, 1673) were mainly interested in nautical matters for the practice of seamanship. It was the Jesuit Père Paul Hoste [41], 1697, who first took on the challenge of estimating from lines plans the displacement (not unlike Bourne and Deane) and a measure of stability. Unfortunately his stability analysis was flawed because he misinterpreted Archimedes and missed the effects of the volume shifts due to a heeling inclination.

In the Netherlands the first pioneering scientific work on ship hydrostatics was performed by the Flemish/Dutch scientist Simon Stevin (1548-1620) to be addressed in more detail in the next section. His work has contributed to an early intuitive physical understanding of the principles of hydrostatics. The Dutch mathematician Johannes Hudde, (1628-1704) had proposed a method in 1652 [42], later called the difference in drafts method, for measuring the cargo payload capacity or tonnage as a basis for port fees and taxes by taking the difference between the displacement of the ship fully loaded minus the displacement empty. Offsets were taken for the waterlines in both loading conditions and the volume between the two waterlines was approximated numerically by means of trapezoids and triangles.. This volume was converted to weight by the Principle of Archimedes. Other treatisers, in particular Witsen [43], 1671, and Van Yk [44], 1697, pursued similar paths, especially for estimates of volume, displacement and payload capacity.

This short survey has been confined to traces of increasing Archimedean influence in ship hydrostatics. Many more details on the work of the treatisers are found in Barker [45] and Ferreiro [46].

In summary it is fair to state that by 1700 Archimedes' text in OFB was known to scientists, but very little of his knowledge had found its way into ship design practice. However in this period the issues of a continuous hull form definition by ship lines plans and of volume and centroid estimates for ship hulls by numerical approximation had been brought to satisfactory practical solutions.

3.5 Stevin, Galileo, Huygens

The rebirth of hydrostatics in the 17th c., directly based on Archimedes' written work in OFB, and its extension to new foundations and applications is essentially owed to the work of three famous physicists, Simon Stevin (1548-1620), Galileo Galilei (1564-1642) and Christiaan Huygens (1629-1695). They thoroughly studied and understood Archimedes' work, especially his treatise OFB, and were able to apply and extend it to practical applications in ship design. Blaise Pascal (1623-1662) achieved the equivalent in aerostatics.

Simon Stevin, the famous mechanicist, astronomer and hydraulic engineer, worked on several fundamental problems of mechanics and also reestablished hydrostatics. He introduced the concept of hydrostatic pressure, which the Greeks had not known. He axiomatically developed a body of propositions embracing the entire fundamentals of hydrostatics in his treatise "The Elements of Hydrostatics" [47] (1586 in Dutch, 1608 in Latin translation). His premises are tantamount to the Archimedean properties of the liquid. In a liquid at rest the hydrostatic pressure increases linearly with the depth in proportion to the specific weight of the liquid since the weight of the liquid column on top of a given point causes this pressure. This opens the door to treating hydrostatics as a special case of field theory in the context of continuum mechanics, as it would be regarded later. Where Archimedes had dealt only with force resultants, Stevin was able to discuss hydrostatic phenomena as the result of pressure distributions. For ships the results are the same, but the approach is different. Stevin in his work gave full credit to Archimedes whom he praised.

Unfortunately Stevin had misunderstood Archimedes' criterion of stability. By disregarding the effects of the volume shift in the heeled position of the vessel from the emerging to the immersed side he came to the erroneous conclusion that for a stable ship the center of buoyancy B must always lie above the center of gravity G. This is actually a sufficient, but not a necessary condition for upright stability.

Galileo, famous as a physicist and astronomer, also occupied himself with hydrostatics, which is not so widely known. In Florence in 1612 he published a treatise "Discourse on Bodies in Water" [48], where he defends his Archimedean viewpoint on the cause and magnitude of the buoyancy force against an opposition of Jesuit clerics who held the Aristotelean scholastic position that bodies specifically heavier than a liquid need not sink in that liquid. According to Aristotle, whether a body specifically heavier than a liquid sinks or floats on the surface depends on its shape. Galileo refuted this view and defined buoyancy in the manner of Archimedes.

Christiaan Huygens in 1650, thus at the tender age of 21 years, fully understood Archimedes' treatise OFB and was able to reconfirm Archimedes' results for the stability of simple shapes (sphere and paraboloid) and to extend the stability criterion to other homogeneous solids, the cone, the cylinder and the parallelepiped [49]. He systematically varied the specific weight and the aspect ratios of these shapes, the main parameters for stability properties, and tested the stability of several shapes for a full circle (360 °) of initial positions. Many solids turned out to have more than one stable equilibrium positions. Huygens used an approach based on the principle of virtual work to define equilibrium, which is equivalent to and thus reconfirms Archimedes' results derived from force and moment equilibrium. He did not apply his methods to ships because he did not have a suitable, continuous analytical hull form definition.

Huygens did not want his three-volume treatise to be published, for he did not consider his results to be complete, useful and original compared with Archimedes. Rather he wanted his manuscript to be burnt. But it was much later found in his legacy and at last published in 1908 in his Collected Works.

3.6 Calculus

To approximate the areas, volumes and centroids of simple shapes Archimedes had used the Method of Exhaustion, usually attributed to Eudoxos. This method relies on small, but finite, not infinitesimal elements to represent curves and surfaces. The method is not directly applicable to ships as objects of arbitrary shape. It defines its geometries by a finite number of polygonal or polyhedral elements. Integration by infinitesimal calculus by contrast is based on a limiting process to an infinite number of polygonal elements and derives its results analytically, often by means of a summation of an infinite series. Thus calculus can be applied to any analytically defined shape, hence also to arbitrary ship shapes.

The invention of calculus has many precursors and contributors (cf. Boyer [50]). But consistent foundations and a well defined methodology were at last developed by Newton and Leibniz in the late 17^{th} c. These methods spread fast during the first few decades of the 18^{th} c. Thus by about 1730, when two leading scientists, Pierre Bouguer (1698-1758) and Leonhard Euler (1707-1783), simultaneously and independently, embarked on addressing the problems of ship hydrostatics again in a modern way, they had the mathematical methods and computational tools available to reformulate the integral quantities in Archimedes' approach in terms of the elegant and precise notation of calculus. Analytical, graphical and functional representation of ship hull shapes were now achievable so that a modern reformulation of Archimedean hydrostatics as an application of continuum mechanics to ships had become feasible.

3.7 Bouguer and Euler

In 1727 the Parisian Royal Academy of Sciences held a prize contest on the optimum placement of masts in a sailing ship. Both Pierre Bouguer, a member of the academy, and Leonhard Euler, a Swiss citizen, then 20 years old and under the tutelage of Johann Bernoulli, participated in the contest and submitted contributions. The subject is closely related to ship hydrostatics because the optimum placement of sail area is a direct function of the permissible heel angle of the ship, where heeling moment and restoring moment are in equilibrium. But in their papers submitted for the competition neither Bouguer nor Euler indicated any knowledge of Archimedean hydrostatics. Bouguer won the award nonetheless and Euler's treatise was acknowledged as noteworthy by an "Accessit" verdict.

Neither scientist seems to have been satisfied by this intermediate result, for both of them continued to work intensively on ship hydrostatics during the next decade and a half, separately and independently without knowledge of the other's results before they were published (Nowacki, Ferreiro [51]). Bouguer participated in a scientific expedition to the Andes in Peru, today Ecuador, for geodesic measurements near the equator from 1735 to 1744 (Ferreiro [52]). He worked on his fundamental ship hydrostatics treatise "Traité du Navire" [53] mainly during this period. It was published in 1746 soon after his return to France. Euler worked on the same subject as a member of the Russian Academy of Sciences in St. Petersburg from 1737 to 1741. This work resulted in his two volume treatise "Scientia Navalis" [54], which after a long delay in the publishing process appeared in 1749. Both scientists respectfully acknowledged the other's work, which had led to largely equivalent results. They confirmed they had not known the other's work prior to its publication.

Bouguer does not mention or give credit to Archimedes anywhere, but treads firmly in Archimedes' footsteps everywhere. E.g. in Book II, Section I, chapter 1 of [53] he introduces the buoyancy force with an explanation that is tantamount to the Principle of Archimedes:

"The principle of hydrostatics, which must serve as a rule in this whole matter and which one must always have in mind, is that a body that floats on top of a liquid is pushed upward by a force equal to the weight of the water or liquid whose space it occupies".

Bouguer in another section also reconfirms this result by integration of the hydrostatic pressure over the submerged surface of the hull.

Euler freely acknowledges the debt he owes to Archimedes for the fundamentals of hydrostatics in buoyancy and stability. Euler begins his treatment of ship hydrostatics with the following axiom in the spirit of Stevin and field theory:

"The pressure which the water exerts on a submerged body in specific points is normal to the body surface; and the force which any surface element sustains is equal to the weight of a vertical water column whose basis is equal to this element under the water surface".

All other results in ship hydrostatics can be derived from this axiom. Euler proceeds to deduce the Principle of Archimedes by pressure integration over the submerged part of the hull surface.

Regarding ship stability Bouguer and Euler went distinctly different ways, both departing from Archimedes' reasoning and both arriving at equivalent results, though expressed in different formulations.

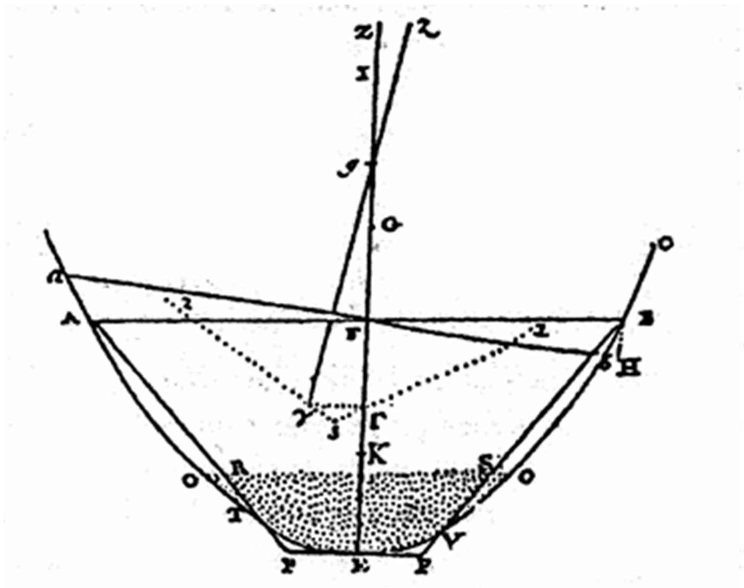

Fig. 5: Bouguer’s Derivation of the Metacenter (from [53])

For initial stability, i.e., the stability in the initial upright condition for very small (infinitesimal) angles of heel, Bouguer introduces the metacenter (point g in Fig. 5) as the point of intersection of two infinitesimally neighboring buoyancy directions. In Fig. 5 in the upright condition the hydrostatic pressure resultant or buoyancy force acts through the volume centroid Γ of the submerged volume, also called center of buoyancy B in modern terminology. As the ship is slightly heeled, a slice of volume whose cross section is triangular in the figure is moved from the emerging side to the immersed side so that according to Archimedes’ shift theorem (law of the lever) the center of buoyancy moves parallel to the shift from Γ to γ. The new buoyancy direction runs normal to the new liquid surface and intersects the upright line of buoyancy in point g, the metacenter. The metacenter is at the same time the center of curvature of the curve that is the locus of the centers of buoyancy as the ship heels continuously. Bouguer constructs the metacenter by means of calculus and numerical integration (trapezoidal rule). His stability criterion is that the center of gravity of the ship (point G in Fig. 5) must not lie above the metacenter g. The ship is stable for positive metacentric height Gg.

Euler likewise begins with an inclined, heeling ship (Fig. 6) in analogy to Archimedes. He then applies the shift theorem to the emerging and immersed volume elements and constructs the new location of the center of buoyancy by integration of cross-sectional data over ship length using a calculus formulation. This enables him to compute the restoring moment, i.e., the couple formed by the buoyancy and gravity forces in the inclined position. If a positive restoring moment (or positive righting arm, similar to Archimedes) is acting, then the ship tends to restore itself to the upright condition after a small heeling disturbance, hence is stable.

For infinitesimal angles of heel the metacentric approach by Bouguer and the restoring moment criterion by Euler lead to identical expressions. For finite angles of heel both scientists have proposed valid methods and criteria, though in different form. Both can claim that they have achieved a stability analysis based on either the metacentric curve or on the restoring moment/righting arm, which can be implemented by calculus and numerical integration. Both results are extensions to Archimedes’ theory applicable to ships of arbitrary shape.

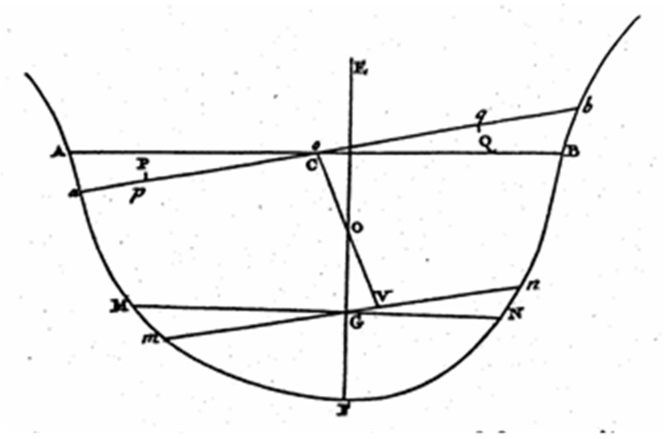

Fig. 6: Euler's Centroid Shift in Inclined Cross Section (from [54])

Both Bouguer and Euler have also addressed stability criteria for finite angles of heel, viz., the metacentric curve or the restoring moment in the inclined position, respectively. Further they both demonstrated how to deal with longitudinal stability, i.e., the response of the ship to trimming moments, viz., the longitudinal metacenter or the longitudinal restoring moment and the trim angle. They proceeded to show how many other applications in ship design and operations can be treated once the basic hydrostatic stability responses of the ship are known. This includes calculations for the determination of the draft, heel angle and trim angle of the ship for any loading condition, e.g., during the loading and unloading of the ship; ship motions under wind load and in waves, maneuvering dynamics under sail and much else. Calculus formulations and their numerical evaluations paved the way toward practical application of hydrostastic analysis of ship performance in general.

Bouguer's and Euler's published fundamental treatises in ship theory experienced quite contrasting reception and distribution. Bouguer's French text was readily understood and illustrated by many numerical examples. Textbooks for colleges were soon prepared in France on its basis. The French Navy soon made stability assessment by the metacenter criterion an official requirement for any new design. Euler's Scientia was written in Latin, lacked numerical examples, did not reach many practitioners and remained relatively unknown in shipbuilding practice. But it was recognized as a valuable reference in future scientific work.

3.8 Chapman and Atwood

Soon after the scientific foundation for calculations of the floatability and stability of ships had been laid by Bouguer and Euler, the first attempts were made to apply this knowledge in practical ship design. Frederick Henrik Chapman (1745-1807) in Sweden and Thomas Atwood (1745-1807) in Britain were two outstanding engineering scientists and designing naval architects who took advantage of this new knowledge and adapted it to their practical needs. This brought the physical insights of Archimedes to fruition in practical design for the first time in full scope after a long delay of about two millennia.

Chapman, the son of an English shipbuilder and immigrant to Sweden, grew up in an atmosphere of practical shipbuilding orientation and scientific openness. As a young man he was both practically trained and mathematically oriented, so he picked up a broad basic education. He also spent a few years in England, France and the Netherlands in a sort of self-paid shipyard traineeship and thus became familiar with not only the practical skills of the trade, but also with the recent scientific knowhow in those leading shipbuilding nations. He learnt about the work of the Bernoullis, Bouguer and Euler and knew how to apply it in his own ship design work. Thereby he was firmly entrenched in the tradition of Archimedes. He returned to Sweden in 1757 and soon acquired much responsibility in Swedish naval and merchant ship design, rose to high rank and remained in a leading position in shipbuilding throughout his lifetime. He was thereby able to test his basic new insights and methods in practical design work and shipbuilding. Cf. Harris [55] for detailed biographical information.

He also took pleasure and pride in publishing his insights and practical methods in treatises of technical and scientific orientation, very suitable as texts for ship design education. His "Treatise on Shipbuilding" [56], 1775, stands out as a textbook on his design methodology. He applied his knowledge of Bouguer's and Euler's work and implemented numerical quadratures by Simpson's rule, having taken private lessons from Thomas Simpson in England. Chapman also was an excellent designer of ship lines plans. Reportedly he drew some 2000 lines plans in his professional career. Many of these were documented in print and published. He made it a routine to calculate the displacement and a stability measure, the metacentric height, for every ship. Chapman also estimated the wind loads on the sails for critical operating conditions in order to provide sufficient safety margins in metacentric height for stability.

Chapman also knew how to influence hull shape and centroid location by geometric variation in order to set favorable stability indices (like metacentric height), viz., enough stability to be safe against extreme heeling moments, but not too much in order to avoid abrupt ship motions in rough sea states. This illustrates how a stability analysis based on Archimedes' method was fully integrated into the design process.

George Atwood (1745-1807), an English mathematician and physicist, together with his French partner, the naval constructor Vial de Clairbois, recognized that the initial stability at small angles of heel was not sufficient to ensure a ship's safety, as Bouguer and Euler had already pointed out. Thus they investigated the ship's righting moments at finite angles of heel, as Archimedes had done for the paraboloid. They used numerical quadrature rules to calculate the "righting arm" of the ship for a given draft, center of gravity and heel angle over a wide range of heel angles (cf. [57]).

Thus by the end of the 18th c. all prerequisites were available to perform a complete displacement and stability analysis for a ship in its design and any other loading conditions. The application of Archimedes's knowledge was thereby extended to actual ships.

4. Archimedes in Modern Ship Design

4.1 Scope

The hydrostatic principles of Archimedes govern the floatability and stability of ships, two crucial elements of ship safety. Safety considerations pervade the entire ship design process. Thus the principles of Archimedes are also deeply embedded in the modern ship design process.

All areas of ship design are interdependent and thereby closely connected with each other (Fig. 7). They all contribute to the overall efficiency and safety of ship performance. At the same time decisions in one area influence the others. The modern design process is viewed as an integrated decision process and is judged by the overall performance in all categories.

Safety is a design target in its own right. Safety is not confined to Archimedean floatability and stability requirements. Rather it encompasses all aspects of hazardous secenarios in the ship's lifetime. This must hold for all operating and loading conditions of the ship, also when the ship is damaged and perhaps partially flooded. Safety in ship operations aims essentially at the protection of human lives, at the prevention of damages to or loss of property objects, including the total loss of the ship, and at the protection of the natural environment of the ship. Safety hazards may jeopardize any or all of these vulnerable goods. Thus design considerations must account for all known hazards, evaluate their risks, strive for prevention measures and seek an adequate safety margin in all categories.

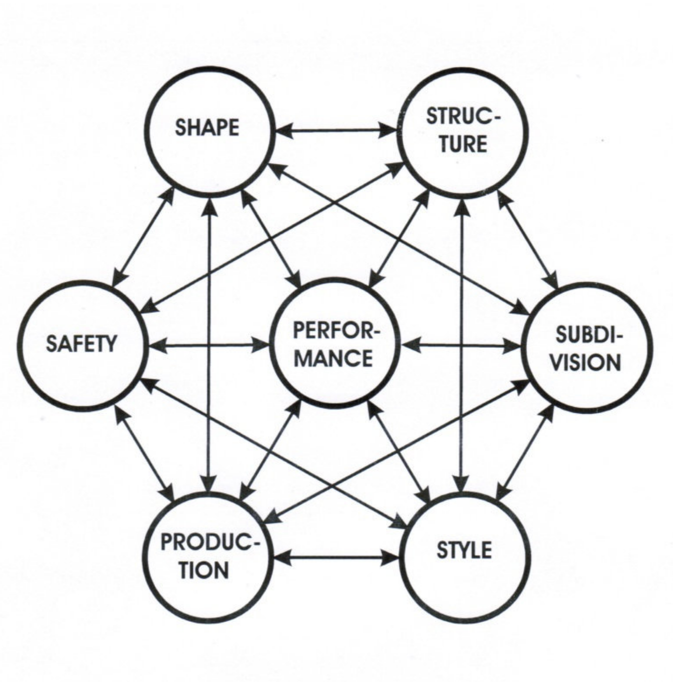

Fig. 7: Structural Elements of the Ship Design Process

In modern Risk-Based Ship Design this is performed by aiming at the most adequate, achievable combined risk for all hazards in a most cost-effective manner. This will be discussed in more detail later.

Safety performance and shape design strongly interact. As for the principal dimensions, e.g., a shorter beamier ship of a given displacement tends to be more stable, though usually at the expense of a greater resistance, hence drawbacks in speed and power. Thus there are tradeoffs between two important performance measures, a frequent situation in design. Therefore it is welcome that both of these effects can be quantified in design stage calculations, where Archimedes helps in modern design. The management of centroid locations and metacentric height by hull form changes is another example of interaction between safety and shape design.

Structural integrity, especially in collision or grounding scenarios, is another concern for safety. The watertight subdivision of the ship, when damaged and partially flooded, is crucial for the survival of the ship and the humans aboard and for the protection of the environment. But such compartmentation is only adequate, if the ship remains floatable and stable in the damaged condition. This was a bitter lesson from the sad Titanic disaster in 1912.

Matters of style, e.g., the layout and placement of decks and spaces between keel and superstructures, may much influence the stability and damage safety performance of a ship, as some sad accidents have reminded us. Methods of production enter into the reliability of the structure (safety), but also weight distribution and centroid location (materials).

In summary, practically all essential areas of design interact with the safety parameters of the ship and must be monitored and controlled during the design process. This must be done quantitatively at the design stage by keeping track of the ship's floatability and stability in all operating conditions, especially after damaging accidents. Thus in modern times no certified ship can be designed without resorting to the principles and criteria of Archimedes.

4.2 Developments and Trends in Ship Design for Safety

- **Overview**

Around 1800 a new era in shipbuilding and shipping was about to begin with the advent of steam powered steel ships. It was a fortunate coincidence that by this time the scientific knowhow in ship safety matters had reached sufficient maturity to be applied to this new generation of ships. In the past the technology of wooden sailing ships had been in gradual evolution for many centuries so that new designs could be based on the experience with existing and past ships. But the degree of innovation in the new ships that arrived with the industrial revolution was far too radical to base the safety concepts on inadequate experience alone. Thus science and technology together had to arrive at new methodologies to ensure safety in ship design.

The transition from wooden sailing vessels to steam powered steel ships, which lasted more than a century, brought along many subsequent changes with new technologies and new risks. Ships grew in size and speed for greater economy, but were also equipped with an increasing number of auxiliary systems to improve their safety. New specialized ship types were developed for new commodities, e.g., tankers for oil, later gas and chemical transport. The transoceanic passenger trade gained much momentum from the 19th into the 20th century and created a large international fleet of fast, often luxurious passenger liners. This rapid diversification in shipping tasks and growth in seaborne trade volume generated many new technical challenges and operating risks. Simultaneously the value of the larger, faster ships and of their payload commodities grew immensely, hence also the economic risks of shipping investments.

- **Classification Societies**

Ship owners and insurance companies consequently were interested in assessing such risks and finding technical solutions to control them during the continuing process of technical innovation. They needed advice in technical and scientific expertise from impartial, qualified maritime experts familiar with shipbuilding and shipping practice and with the scientific state of the art. Such services from the beginning of the steel ship era to this day were provided by classification societies in several leading maritime countries.

Classification societies were established, mainly during the 19th c., as national bodies and as legally independent, non-governmental organizations to promote the standards of safety at sea in close cooperation with the maritime industries. They perform their functions by developing and publishing classification rules for the design, production and operation of ships (and other offshore structures) by surveying the design process for compliance with the rules, by issuing classification certificates to approved ships and by periodic inspection of the ships in operation. Their certificates are the basis for obtaining marine insurance contracts.
A core activity of classification has always remained the promotion of safety in the areas of ship strength, ship stability, load lines and ship subdivision for damage control. Thereby and through their International Association of Classification Societies (IACS) they have much contributed to the national and international ship safety legislation. Their activities remain consultative since they have no executive power in the marine industry.

- **National and International Rules and Conventions: The Load Line**

Ship safety legislation was initially based on national, sometimes even local, rules and regulations, which were slow in gaining ground. Regarding the load line of ships it was not before 1876 in England that a law was passed requiring a mark (the Plimsoll mark) to be placed on the vessel's sides to prevent overloading the ship or to ensure a minimum freeboard. Sufficient freeboard is necessary to provide adequate reserve floatability (against sinking due to excessive heave motions) and reserve stability (against capsizing due to large heeling moments). Grave accidents initially occurred with low freeboard vessels until legislation recognized the necessity of reserve buoyancy above the load waterline. National rules set a trend, but it took until 1930 before the first International Load Line Convention was passed.

National legislation was insufficient to secure uniform safety precautions in international shipping. Conventions to prepare international agreements were long desired. It took until 1948 that a decisive step was taken. The United Nations in 1948 inaugurated an international convention, first called Inter-Governmental Maritime Consultative Organization (IMCO), later in 1982 renamed International Maritime Organization (IMO), to be concerned internationally by cooperation between governments with matters of maritime safety *to encourage and facilitate general adoption of the highest practicable standards in matters concerning maritime safety, efficiency of navigation and prevention and control of marine pollution from ships* (cf. Lamb [58]).

The International Load Line Convention of 1930 was revised in 1966 and again in 2000 to account for the ship's seakeeping dynamics under the management of IMCO/IMO.

- **Damage Stability**

The catastrophic accident of the Titanic in 1912 with the loss of many human lives caused international alarm and drew the attention to be focused on the safety of ships when damaged and partially flooded. An international Convention on Safety of Life at Sea (SOLAS) in 1914 presented new criteria for safety regulations for passenger vessels, but due to delays by two World Wars the results were not passed and put into force until SOLAS 1948 under IMCO supervision. The regulations were gradually amended and stiffened, also in consequence of new grave accidents (SOLAS 60 and 74). Recent developments set safety standards for dry cargo ships in damaged condition (SOLAS 90) and moved on to probabilistic concepts of damage assessment for dry cargo and passenger ships (SOLAS 2009). The purpose of these regulations is ensuring safe design by sufficient subdivision of the hull by watertight bulkheads controlling the size, location and number of flooded spaces so as to control sinkage, trim and heel in the damaged floating condition not to exceed a safe margin for the ship's floatability and stability. The flooded spaces are either regarded as filled with added liquid weight or equivalently treated as lost volumes of buoyancy. In every other regard the analysis of the damaged condition is based on the same Archimedean principles as for the intact condition.

- **Protection of the Environment**

The significant growth of ocean oil transport after WWII and sad accidents with tankers resulting in dramatic oil pollution in the ocean and on shores have caused growing concern over the threat of oil pollution in the maritime environment. This concern was addressed under the auspices of IMO at the MARPOL 73/78 conventions, which went into force in 1983. To limit the potential oil outflow in the event of tanker damages by collision or grounding the MARPOL regulations require from all new tankers of more than 20000 tons deadweight the arrangement of segregated ballast tanks (clean tanks) in protective locations, i.e., to shield the cargo tanks. This has led to new compartment configurations in "double-hull" tankers with clean ballast tanks along the sides and in the double bottom. Such arrangements also result in more potentially empty spaces and hence more freeboard, which adds to the reserve buoyancy. How these reserves can be used to the best advantage of ship safety, has been the subject of recent discussions and design optimization studies (cf. Papanikolaou [59]).

4.3 Risk-Based Ship Design

The design of complex systems operating under hazardous conditions and subject to threats of immense damages in the event of catastrophic failures has become a specialized discipline, now commonly called risk-based design. This approach has been a necessity in the nuclear industry for many decades and has also prevailed in aerospace design and in other industries with great public and economic risks. In the maritime field the offshore oil industry first introduced this approach by legislation based on risk analysis for offshore systems, e.g., in Norway in 1986, in the U.K. in 1992. For ships IMO is currently following a strong trend toward risk-based ship design in the development of new safety standards (cf. Sames [60], Skjong [61]).

This entails a number of methodical elements:

- Future standards and some current pilot regulations are intended to replace, at least in part, the traditional rule-based approach of classification and regulations, which describes in technical detail how a safe design is to be realized, by Goal-Based Standards, where a safety goal is set regardless of how it will be achieved. This requires Quantitative Risk Analysis (QRA) with quantified risk assessment. Goal-Based Ship Design (GBSD) aims at an optimal solution for the overall safety of the ship. This is to be achieved in the most cost-effective manner.
- The risks will be defined for each hazardous operational scenario in probabilistic terms by the predicted probability of occurrence of the hazardous event multiplied by the economic value of the consequent damage. All damages, whether to the public, the ship owner or to individual humans, are to be included in the analysis. The total risk is evaluated by combining the risks of all scenarios. The total risk will be compared to the acceptable risk, chosen either relative to ships designed by existing IMO rules or in absolute terms based on forthcoming new IMO risk acceptance criteria (cf. Sames [60]).

- In optimizing designs simultaneously for their economic viability and their safety, safety is no longer regarded as a rule-based constraint, but is treated as an objective in its own right. After all, the owner's and the public's interest lies in both economy and safety. Risk analysis quantifies safety targets in comparable units as the functional economic measures.

These features in the risk-based methodology set an ambitious scope for design studies. Yet, IMO discussions and pilot studies are well underway. Many details can be found in the recent book "Risk-Based Ship Design" (Papanikolaou [59]), which is in large measure a result of the EU funded research project SAFEDOR. It contains example studies for a cruise ship (Vassalos [62]) and a double hull tanker (Papanikolaou [63]).

- Formal Safety Assessment

A systematic methodical approach that is often followed in Risk-Based Ship Design is the Formal Safety Assessment (FSA). It is performed in five steps:

- Identification of all relevant safety hazards
- Quantification of the risks for each hazard
- Enumeration of the design options

- Cost-benefit analysis of all design options, including the effects of all hazardous scenarios
- Systematic comparison or optimization studies to recommend the chosen design

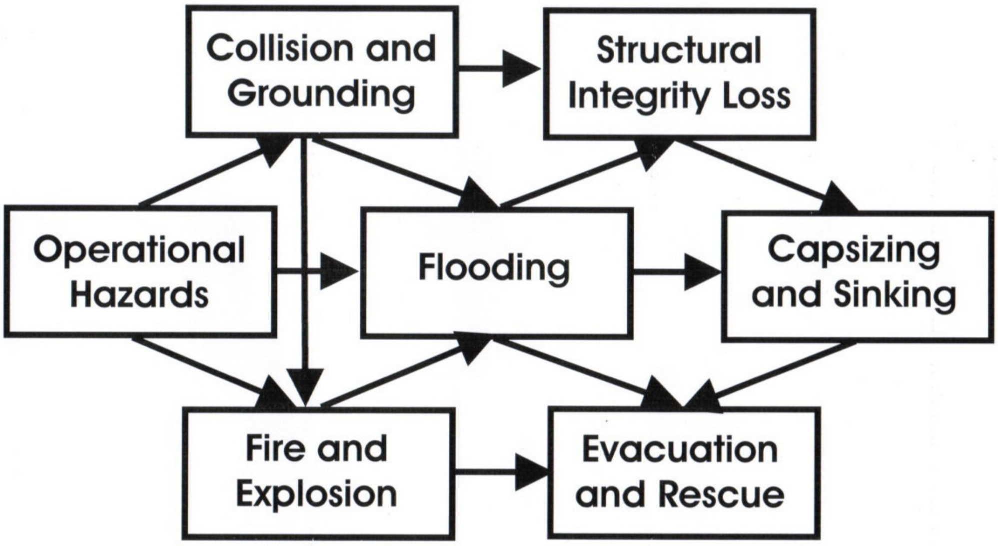

Fig. 8: Structure of Hazardous Scenarios in Ship Safety
(adapted from Vassalos [62])

Fig. 8 shows how the hazardous scenarios in ship safety are connected and depend on each other. The total risk analysis is probabilistic and must account for these interdependencies.

Interesting case studies have been performed (cf., e.g., [62], [63]). Papanikolaou and his team designed a double-hull AFRAMAX tanker, where the reference vessel was an existing rule-based design. Using a risk-based approach it was possible to demonstrate in a multiobjective optimization study that the best goal-based designs, without changes in principal dimensions and hull form, varying double-hull tank dimensions and compartmentation, allow increases in cargo capacity and improvements in environmental safety, viz., reduction in oil outflow according to MARPOL regulations, without drawbacks to the economic performance.

4.4 Trends in Ship Design for Safety

To summarize the major developments in ship design safety, which have accompanied the rise of modern shipping, the following long-term and recent trends can be recognized:

- From predominantly experience-based design to design on scientific-technological foundations
- From national to international rules and regulations
- Safety studies advanced from late to early design
- From deterministic to probabilistic design decision models
- From prescriptive rules to goal-oriented methods
- From feasible design to optimal design
- From modeling safety requirements as constraints to treating them as objectives
- From design against individual hazards to Risk-Based Design for overall safety

All of these developments together have contributed to a much enhanced safety in modern ships for a greatly diversified spectrum of ship types. The ideas of the Principle of

Archimedes and of his stability assessment have remained at the core of every modern ship safety assessment.

5. Conclusions

Archimedes laid the foundations for the hydrostatics of floating systems in his treatise OFB. He based his deductions on experiments of thought and very few physical axioms amounting to the equilibrium of forces and moments applied to an object floating in a liquid at rest. The Principle of Archimedes holds for objects of arbitrary shape, hence also for ships, and his stability criterion of positive righting arms can be extended to solids of non-homogeneous mass distribution like ships.

However it is unlikely that Archimedes was involved in the stability analysis of contemporary ships built in Syracuse though he may have assisted in other ways, but evidently he did not have available continuous ship hull form definitions, methods of integration for areas, volumes and centroids for arbitrary shapes, and any data drawn from experience on safe margins for external heeling moments in critical scenarios.

Despite the initial lack of this further information Archimedes' insights were recognized throughout late antiquity and the Middle Ages as physical fundamentals by those few who had access to his work and they may have been guiding principles in qualitative assessment of design decisions. Prominent scientists, e.g., Stevin, Galileo, Huygens, Pascal, and practitioners in their treatises made elementary contributions in order to make Archimedes' concepts applicable to ship design.

But it was only after continuous representations of ship hull forms, at least graphically by drawings, and methods of integration by calculus had become available by about 1700, that direct numerical application of Archimedes' laws could be brought to bear on practical ship design in volume and stability analysis. We owe Pierre Bouguer and Leonhard Euler the decisive scientific steps toward quantitative, scientifically founded numerical evaluation of ship hull form and stability properties in practical design. Chapman and Atwood with Vial du Clairbois in the late 18th c. are early witnesses of the practical use of such safety relevant calculations at the ship design stage.

The new era of iron ships, propelled by steam, and further rapid innovations in shipping during the industrial revolution created many new ship types and shipping scenarios which required a new, much broader approach to ship safety. A risk-based approach, integrating all relevant hazardous scenarios in a total risk analysis, has gradually matured and is entering into ship design practice. A core element in this modern comprehensive ship safety analysis has remained the assessment of hydrostatic ship properties by Archimedean principles.

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433 Lorraine Daston & Jürgen Renn (Hrsg.) **Festkolloquium für Hans-Jörg Rheinberger**
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434 Conference **Epistemology and History. From Bachelard and Canguilhem to Today's History of Science**

435 Jens Høyrup **Sanskrit-Prakrit interaction in elementary mathematics as reflected in Arabic and Italian formulations of the rule of three – and something more on the rule elsewhere**

436 Jens Høyrup **A hypothetical history of Old Babylonian mathematics: places, passages, stages, development**

437 Jürgen Renn **Schrödinger and the Genesis of Wave Mechanics**

438 Pietro Daniel Omodeo **L'*iter* europeo del matematico e medico scozzese Duncan Liddel**

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441 Horst Kant und Jürgen Renn **Eine utopische Episode – Carl Friedrich von Weizsäcker in den Netzwerken der Max-Planck-Gesellschaft**

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