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Abstract

There has been much speculation about what role the leading German physicist of the Nazi era, the prodigious Werner Heisenberg (1901-1976), played in the failure of Nazi Germany to pursue development of an atomic bomb. Reading extreme views of Heisenberg that appeared in the literature piqued our curiosity, and we analyzed Heisenberg's 1939/40 report to German Army Ordnance hoping to establish additional facts that would throw light on this controversy.

When we read Heisenberg's 1939/40 report, we saw that while it contained many equations that related to the development of a nuclear reactor, it had no mathematical/scientific derivations related to the development of an atomic bomb, although, as we show in this paper, it would have been easy for Heisenberg to develop and supply such information. He actually did it quickly while interned in the U.K., at Farm Hall after the war in Europe ended, incorporating the theory and its implications in his August 14, 1945, lecture.

Nevertheless, despite the fact that Heisenberg did not develop a theory of nuclear explosives, he should have been able to deduce the critical radius (mass) formula for an atomic bomb from information contained, although somewhat scattered, in his 1939/40 report in which he only developed a theory about the workings of nuclear reactors. Not only that, but an estimate of the critical mass, although a crude one along the lines Frisch and Peierls pursued, could also have been obtained that would have suggested that building a nuclear explosive might be feasible. Heisenberg, whose intuition was legendary, should certainly have been able to build on his work on the theory of nuclear reactors to take the steps needed to extend that theory to a nuclear explosive.

We address also the apparent discrepancy between the above statement and Heisenberg's references to "tons" when he spoke of the critical mass at Farm Hall and Heisenberg's short-cut formula that he used in his Farm Hall lecture on August 14, 1945, without any explanation.

Part I: Heisenberg & The Critical Mass: Resolved

Motivation for this Research

What originally piqued our interest in our subject was the extreme contradictions implicit in various authors' views of Heisenberg. In 1998, Paul Lawrence Rose, in Heisenberg and the Nazi Atomic Bomb Project stated that[1] "*it cannot really been doubted that had Heisenberg been able to produce an atomic bomb, he certainly would have done so,*" whereas in 1993, Thomas Powers, in Heisenberg's War - the Secret History of the German Bomb concluded:[2] "*But Heisenberg did not simply withhold himself, stand aside, let the project die. He killed it.*"

Suspecting that we might find information that would help us resolve at least part of the Heisenberg mystery in Heisenberg's publications, we began our research by examining and analyzing his comprehensive report to German Army Ordnance in the early days of World War II.

Heisenberg's report to German Army Ordnance was issued in two parts: "Die Möglichkeit der :technischen Energiegewinnung aus der Uranspaltung," (English translation:[3] "The possibility of the technical acquisition of energy from uranium fission") dated Dec. 6, 1939, and "Bericht über die Möglichkeit der :technischen Energiegewinnung aus der Uranspaltung," (English translation: "Report about the possibility of the technical acquisition of energy from uranium fission") dated Feb. 29, 1940.

We read this report, which was declassified in 1971, in the book Werner Heisenberg's Gesammelte Werke / Collected Works, Series A/Part II, Original Scientific Papers / Wissenschaftliche Originalarbeiten, Walter Blum et al., eds. (Berlin: Springer-Verlag, 1989), pp. 378--418 (Ref. [4]). The first part (pp. 378-396, also referred to as G39, see p. 375) contains the basic theory. The second part (pp. 397-418, also referred to as G40, see p. 375) contains more specific calculations and evaluation of measurements.

The Flaws in Rose's Technical Findings

When we read Rose's *Heisenberg and the Nazi Atomic Bomb Project* (Ref. [13]), we were fascinated by the technical arguments associated with the critical mass problem, where critical mass is the amount of uranium needed to produce an atomic bomb, normally expressed as the critical radius of a sphere of uranium. This topic came up in a private conversation between Otto Hahn and Heisenberg at Farm Hall on Aug. 6, 1945, the day the atomic bomb was dropped on Hiroshima. Heisenberg used the so-called random walk (RW) model to illustrate to Hahn how an atomic bomb works. Such a model yields a requirement of tons for the critical mass (a critical radius of 54 centimeters) instead of the few kilograms actually needed. But about one week later Heisenberg gave his fascinating lecture on Aug. 14, 1945, in which he presented an entirely different approach -- resulting in a figure of 6.2 centimeters for the critical radius.[4]

1 [13], p. 269.
2 [11], p. 479.
3 Translation from Ref. [7], Cassidy's *Uncertainty*, p. 421.
4 [2], p.178.

Rose presents a scenario in which he portrays Heisenberg as one who is unable to recognize the correct concept of critical mass during all of the war years and then able, by a stroke of genius, to provide the correct solution after a little more than a week's work. It must be recognized that Rose, in effect, moves Heisenberg's thinking, based on the RW model remark, back in time to 1939/40. To justify this, he quotes Heisenberg's 1939/40 (G39) report to German Army Ordnance and claims that Heisenberg made a significant error.

Heisenberg's critical mass blunder even found its way into the popular press, e. g., on March 27, 2005, when The New York Times wrote: "*In 1939, the German physicist Werner Heisenberg apparently made a mistake when he tried to calculate how much uranium would be required to build an atomic bomb and got a number way too high, discouraging German bomb makers from pursuing a bomb made of uranium, and perhaps stave off an apocalypse.*"

This was, indeed, a fascinating story -- too good to be true?

Before going further we quote Heisenberg's definition of the $\frac{n'}{n}$ ratio which Rose referred to below:[5]

$$\frac{n'}{n} = \frac{\text{number of escaping neutrons/second}}{\text{number of neutrons generated by the neutron source/second}} \,. \tag{1}$$

And the formula Heisenberg developed for a uranium sphere surrounded by a tamper, expressed in terms of the radius, R, of the uranium sphere, and the diffusion length, l, reads:[6]

$$\frac{n'}{n} = \frac{1}{\cos(R/l)} \tag{2}$$

With the critical radius[7] $R_c = \left(\frac{\pi}{2}\right)l$ (see Eq. (13)), it follows that

$$\frac{n'}{n} = \frac{1}{\cos[(\pi/2)(R/R_c)]} \tag{3}$$

Note that $\frac{n'}{n}$ becomes infinite at $R = R_c$.

Rose believed that he had found the underlying reason that Germany did not succeed in building an atomic bomb by "proving" that Heisenberg did not grasp the concept of critical mass. Rose concluded that[8] "*In accordance with established views from 1939 on, Heisenberg regarded critical mass as defined by the balance of escaping and internally produced neutrons; if the internal neutrons exceed those escaping, then the critical mass is realized and a chain reaction is initiated.*" Heisenberg did not

5 [4], pp. 388 and 390.
6 [4], p. 390, Eq. (45).
7 Heisenberg's symbol R_k was replaced with R_c for editorial reasons.
8 [13], p. 217.

define critical mass that way, however, and he did not mention internally produced neutrons either. But he considered the number of neutrons which escape (n') and number of neutrons produced by a neutron source (n). This ratio (see Eq. (3)) becomes infinite at the critical radius. Its reciprocal $\frac{n}{n'}$ therefore becomes zero. Rose, however, claims that this ratio should be large, stating[9] " … , *in the balance definition of critical mass that appears in Heisenberg's G-39 of December 1939, represented as "n/n'," the excess would have to be large. How large would it have to be for a bomb to explode? To find a solution to the bomb problem, Heisenberg had addressed himself to the problem of ensuring that the reaction continued long enough to fission a large number of nuclei -- say, 2^{80} nuclei -- to produce a large explosion. This led Heisenberg in 1940 to conceive of a practical 'critical mass' relevant to a bomb, and he thought of this in terms of an upper limit […] arrived at by means of his random-walk back-of-an-envelope calculation. This is the practical 'critical mass' for exploding 2^{80} nuclei without 'wasting a neutron.' It may be a theoretically true construct, but it scarcely represents accurate or scientific or serious scientific thought about a bomb. Nevertheless, Heisenberg was content to let his thinking rest on this shaky basis for the remainder of the war.*"

This is Rose's key argument, which tries to connect Heisenberg's November 1939 (G39) report to his random walk model. Rose's "proof" breaks down due to the misinterpretation of Heisenberg's $\frac{n'}{n}$ ratio and, in addition, due to his misidentification of "neutrons generated by the neutron source" as "internally generated neutrons."

N. P. Landsman -- in his excellent publication "Getting Even with Heisenberg," not only reviews Rose's book, but also provides a summary of important writings by others -- observes that[10] "*[t]he question of the critical mass of an atomic bomb receives considerable, if not obsessive, attention,*" and[11] "*[b]eyond the issue of the critical mass, Rose gives an interesting account of various ideas on 'reactor bombs', showing that at a certain stage Heisenberg saw a nuclear bomb as an extreme type of nuclear reactor, with highly enriched Uranium and vast quantities of moderator, that went out of equilibrium.*"

We have concluded that Landsman did not check Rose's critical mass claim because he reasonably felt no need to take the time and trouble to pursue the subject to the extent that we have in preparing the present paper. But reading an early version of our analysis, he opined that the information in our paper is worthy of publication.

Our first doubts about Rose's critical mass scenario were aroused when we realized that the RW model is probabilistic in nature and thus does not result in a precise value of the critical radius -- although in the literature the value of 54 centimeters mentioned above is treated as an exact quantity. Note that Heisenberg's RW approach, in a large number of repeated trials, can be imagined as a representation of a walk by a drunkard, taking steps of six centimeters to go left or right with equal chance. The drunkard (generated neutron in a chain reaction) will, with probability of 0.68, be any-

9 [13], p. 218.
10 [10], p. 316.
11 [10], p. 317.

where from the starting point to a distance of 54 centimeters. Choosing a probability of 0.95, the 54 centimeters change to 106 centimeters. Such an argumentation did, in our opinion, not make sense; how likely is it that a reputable scientist would in all honesty suggest such an approach as a design criterion?

Heisenberg, however, employed the RW model in his private conversation with Hahn the day Hiroshima was bombed, as shown in Section "Heisenberg's Puzzling Critical Mass Statements at Farm Hall" below. We explain that Heisenberg needed such a model, which provides a simple explanation of how an atomic bomb works, in order to communicate effectively with Hahn, a chemist, not familiar with nuclear theory.

We address the claim by Rose[12] "*that at a certain stage Heisenberg saw a nuclear bomb as an extreme type of nuclear reactor, with highly enriched Uranium and vast quantities of moderator, that went out of equilibrium*" in the Section "The Reactor Bomb -- Fact or Fiction?" below. We argue that this cannot be true by pointing out the fact that Heisenberg recognized that there is an enormous difference between a nuclear reactor, which requires at most enriched U_{92}^{235},[13] and a nuclear explosive, which requires almost pure U_{92}^{235}. Heisenberg's statement in his 1939/40 report to German Army Ordnance quoted immediately below provides a clear understanding of the underlying physics:[14]

> 1. This explosive transformation of uranium atoms can only take place in almost pure U_{92}^{235}, because neutrons will be absorbed at resonance points of U_{92}^{238} even if only small amounts of impurities of U_{92}^{238} are present.[15]
>
> 2. Enrichment of U_{92}^{235} is the only method to make the volume of the machine small in comparison to 1 cubic meter. Moreover, it is the only method to produce explosives which exceed the explosive power of the strongest available ones by several powers of ten.[16]

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12 [10], p. 316.

13 For details see Section "The Embedded Critical Mass Concept" in "Part III: Hidden Information in Heisenberg's 1939/40 Report" below.

14 [4], pp. 389 and 396. Note that Heisenberg's 1939/40 report is contained on pages 378-418.

15 Original German version (Ref. [4], p. 389): "Diese explosionsartige Umwandlung der Uranatome kann aber nur in fast reinem U_{92}^{235} auftreten, da schon bei geringen Beimengen von U_{92}^{238} die Neutronen in der Resonnanzstalle von U_{92}^{238} weggefangen werden."

16 Original German version (Ref. [4], p. 396): "Die Anreicherung von U_{92}^{235} ist die einzige Methode, mit der das Volumen der Maschine klein gegen 1 cbm gemacht werden kann. Sie ist ferner die einzige Methode, um Explosivstoffe herzustellen, die die Explosionskraft der bisher stärksten Explosionsstoffe um mehrere Zehnerpotenzen übertreffen."

Heisenberg thus expresses his understanding that enrichment for obtaining almost pure U_{92}^{235} is a prerequisite for development of an atomic bomb when he writes: "*This explosive transformation of uranium atoms can only take place in almost pure* U_{92}^{235} ... ." Furthermore, he knew from Bohr's publication at the beginning of 1939, that U_{92}^{235} can fission at low as well as high neutron energy levels (with slow and fast neutrons).[17] Note that chain reactions with fast fission neutrons in the active region consisting of pure U_{92}^{235} uranium can take place because there are no ^{238}U atoms that can absorb them, and in almost pure U_{92}^{235} the ^{238}U content has been reduced to such a level that absorption can be neglected.

With the results shown in this section and the next one as well as in "Part III: Hidden Information in Heisenberg's 1939/40 Report" below, the allegations concerning Heisenberg's incompetence fall apart.

To provide our answer to the question "why did nobody else before you analyze Heisenberg's theoretical work contained in the first part of his 1939/40 report, known as the G39 report"[18] we point out that Meyer (co-author of this paper), after seeing Frayn's drama "Copenhagen," in 2001 became fascinated by the puzzles surrounding Heisenberg and started to write a book about it.[19] And the obvious first step in obtaining more details was to read Powers' *Heisenberg's War - the Secret History of the German Bomb* (which led Frayn to write his drama) and Rose's *Heisenberg and the Nazi Atomic Bomb Project*. The information in Rose's book, as elaborated on in the first three paragraphs of this section, made it clear that the important topics to investigate were Heisenberg's "tons" remarks about the critical mass at Farm Hall (see Section "Heisenberg's Puzzling Critical Mass Statements at Farm Hall" below), Heisenberg's $\frac{n'}{n}$ ratio in the first part (G39) of his 1939/40 report, and his lecture on August 14, 1945 at Farm Hall. The latter made it mandatory to read Jeremy Bernstein's *Hitler's Uranium Club*. This book was tremendously helpful, because it not only contains transcripts of the secretly recorded discussions the German scientists had during their internment at Farm Hall, but also numerous annotations in which Bernstein explains the technical topics under discussion. The information of special interest here are the annotations (#113-273) in English associated with Heisenberg's Farm Hall lecture (pp. 169-190) as well as the version in German (pp. 191-207). With all of this information combined we could now analyze Heisenberg's G39 report in a systematic way. We surmise that without such a broad approach to analyze Heisenberg's work it would be much more difficult to reach the conclusions we did. In other words, just looking at the (G39) report all by itself might not be enough and that could be the reason why, as far as we researched, nobody else published an analysis of Heisenberg's G39 report to German Army Ordnance.

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17 Since Bohr, in the Spring of 1939, published this property of ^{235}U before the start of WW II, Heisenberg knew about it. See Ref. [5] for more details.

18 Below we replace the term "theoretical work ... in his G39 report" with the term "G39 report."

19 Schwarz joined the project that resulted in this paper in 2007.

This analysis was essentially completed in 2010. Missing from it, however, were explanations of apparent contradictions between what the report makes clear that Heisenberg knew -- for example, his awareness that the critical mass was on the order of kilograms -- and things that he said later -- for example, references to tons of ^{235}U -- at Farm Hall. The explanations materialized in the process of completing the paper in 2014 (see Section "Heisenberg's Puzzling Critical Mass Statements at Farm Hall" below). It was then that we made a serious attempt to publish the paper, coordinating it more closely with the anticipated completion of the manuscript of a book by Meyer (co-author of this paper) and Stanly A. Kurzban, tentatively titled *The Puzzles of Heisenberg, and Why There Was No Nazi 'Manhattan Project.'*[20]

The Reactor Bomb -- Fact or Fiction?

To start out with, it is important to know that building an atomic bomb was not on Heisenberg's mind, as Mark Walker in *Nazi Science: Myth, Truth, and the German Atomic Bomb* wrote:[21]

"After he had drafted his two-part report on energy production by means of nuclear fission in the winter of 1939/40, Heisenberg no longer took an active immediate interest in uranium machines. He recognized the great potential of applied nuclear fission, but while he avidly followed the development of nuclear power as an administrator, he also left most of the actual theoretical and experimental research to others. Work on the theory of uranium machines continued at the Kaiser Wilhelm Institute for Physics under the direction of Carl-Friedrich von Weizsäcker, but after he had demonstrated the importance of transuranic elements as nuclear fuel, he lost immediate interest as well. Both Heisenberg and his friend and younger colleague Weizsäcker were concerned with fundamental physical problems, not technical or practical matters. Most of the theoretical research on uranium machines was carried out by two of Weizsäcker's graduate students, Karl-Heinz Höcker and Paul Müller."

Rose asserts that Heisenberg wanted to build a nuclear explosive since early in 1940 and goes to some length[22] to show that the Germans were thinking about a reactor bomb, which Rose explicates as follows:[23] "*Before the scientific principle of the atomic uranium bomb was clearly understood (i.e., that fast neutrons produced in a small critical mass of U_{235} an explosive chain reaction), suggestions were made as to how an explosion could be created [...] by slow neutrons in a reactor stocked with either unseparated uranium or enriched uranium. In Germany research was pursued on the hypothesis that such a reactor-bomb would essentially be an unstable reactor using a moderator and highly enriched U_{235}. This was seen in Germany as a solution to the insurmountable problem of obtaining the tons of U_{235} thought to be required for a U_{235} bomb.*"

To back up this claim, Rose provides the following details:[24]

20 Kurzban joined the project in 2010.
21 [15], pp. 36-37.
22 [13], pp. 115-130, 146-154, and 185-205.
23 [13], pp. xix-xx.
24 [13], p.128.

> "In Berlin, K.-H. Höcker reported during April that he thought a reactor could be built ... and a few weeks later P.O. Müller wrote a paper on *A Requirement for the Utilisation of Uranium as an Explosive* which showed clearly that the Germans' thinking on uranium bombs was still very backward ... there would have to be at least 70 per cent more uranium 235 atoms in the explosive than uranium 238 ... water would have to be present as a moderator; in other words that the explosion would be caused by slow neutrons."

However, from Heisenberg's two statements of requirements, as we describe them in the section "The Flaws in Rose's Technical Findings" above,[25] it follows that he had already concluded in his G39 report that nuclear explosives can be built only with almost pure ^{235}U,[26] and in that case only fast neutrons come into play. Consequently, it is difficult to believe that Heisenberg seriously considered the development of a reactor bomb, which uses at best enriched uranium. Whatever other scientists, e.g., Müller, thought about the concept of a reactor bomb, and whatever research was done (even with Heisenberg's knowledge[27]) in that direction, should not be interpreted as being consistent with Heisenberg's thinking. Heisenberg probably gave the researchers a free hand to pursue their own ideas while he himself spent his time divided evenly between Leipzig and Berlin until 1942.[28] Heisenberg, even if he did not believe in the idea of developing a reactor bomb, might have welcomed Müller's work on the concept as a way of justifying funding for the project by showing that something was being done that higher-ups would believe might result in the development of an atomic bomb. It also served for a time to keep Müller from being drafted into the army. But that happened anyhow, due to the deteriorating state of the war that triggered a wave of call-ups, with tragic consequences:[29] "*Müller left the Heisenberg team only because, like Höcker, he was conscripted for military service in late 1940 or 1941. By the time Heisenberg was able to have Höcker released back to the Kaiser-Wilhelm-Institute in the spring of 1942, Müller was dead.*"

Heisenberg's Puzzling Critical Mass Statements at Farm Hall

In an effort to understand the reasons that may have lain behind Heisenberg's statements at Farm Hall implying that the critical mass is on the order of tons, we distinguish between two time intervals. The first one (T1) is from 6:00 p.m. to 9:00 p.m. on August 6, 1945, the day Hiroshima was bombed and the second one (T2) is after 9:00 p.m. on that same day. During T1, Heisenberg did not believe that the bomb dropped on Hiroshima was atomic, whereas during T2, having heard the details announced in the 9:00 p.m. broadcast, he could no longer doubt its nature.

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25 See the paragraph before and after the indented two paragraphs in Section "The Flaws in Rose's Technical Findings" above.

26 [4], pp. 389 and 396.

27 Translation from Karlsch in *Hitlers Bombe* (Ref. [9], p.73): Heisenberg had already at the start of 1940 asked Weizsäcker, Höcker and Müller to work out the calculations of energy generation in nuclear reactors. As a result, Müller composed in May 1940 a report about *A Requirement for the Utilization of Uranium as an Explosive* (Paul O. Müller, Bedingung für die Verwendbarkeit von Uran als Sprengstoff, G-50, Deutsches Museum München) in which he sketched an instable reactor, later designated as reactor bomb. Karlsch then stated that this concept, which originated under the guidance of Heisenberg, Wirtz und Weizsäcker, is evidence about the deficient understanding of the problems in building an atomic bomb. The reactor bomb was even considered in a patent application.

28 [2], p xxv (Introduction, written by Cassidy).

29 [13], p. 129.

We start with the claim that Heisenberg knew during the war that the critical mass was on the order of magnitude of kilograms, not tons, and we give a reasonable explanation for the apparent discrepancy between this statement and Heisenberg's references to "tons" when he spoke of the critical mass at Farm Hall.

Conclusive evidence that Heisenberg knew during the war that the critical mass was on the order of magnitude of kilograms is the fact that an anonymous report[30] to German Army Ordnance in the spring of 1942 mentioned a critical mass estimate between 10 and 100 kilograms. We agree with Walker[31] that, irrespective of who actually wrote which words, Heisenberg, although uncredited, must have been responsible for this estimate. Supporting evidence is Hahn's remark (also quoted below):[32] "*But tell me why you used to tell me that one needed 50 kilograms of '235' in order to do anything. Now you say one needs two tons.*"

Additional circumstantial (indirect) evidence is presented in "Part III: Hidden Information in Heisenberg's 1939/40 Report" below.

Although the following cannot be considered as authoritatively documented due to the lack of primary sources,[33] it is still worth noting how Heisenberg answered Field Marshal Erhard Milch's question in the Harnack Haus meeting in Berlin on June 4, 1942:[34] "*Field Marshal Milch recalled asking during the meeting how large a bomb would have to be to destroy a large city, such as London. To the astonishment of his audience, Heisenberg reportedly replied, 'about the size of a pineapple' -- perhaps referring only to the U-235 content. The incredulous officials must have thought the scientist insane.*" It is also of note that the German physicist Manfred von Ardenne[35] recalled that Heisenberg told him that only a few kilograms of ^{235}U are needed to trigger a chain-reaction.

Bernstein acknowledges the importance of the 1942 document by writing in his paper "Heisenberg and the critical mass":[36] "*The report, which is dated February of 1942, is called Energiegewinnung aus Uran,* [Energy Extraction from Uranium] *is one-hundred-forty-four pages long, and covers all aspects of the work that had been done since 1939.*" Comparing it with Heisenberg's 1939/40 report, Bernstein remarked that this "*... second wartime report, which appeared in 1942, is even more impressive.*"

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30 [15], p. 172. Because there was neither Uranium 235 nor Plutonium (element 94) available, no accurate calculation of the critical mass was possible. Hence this estimate covers a uranium as well as a plutonium bomb. This is of no consequence to our analysis, since our approximations apply for employing uranium or plutonium as fissionable material.

31 [16], p. 216.

32 [2], p. 118.

33 The only original written documentation of the exchange between Milch and Heisenberg is in the written transcript of David Irving's interview with Heisenberg during the 1960s. Irving included a description of the incident in his book published in England under the title *The Virus House*, London: Kimber (1967), and in the U.S. as *The German Atomic Bomb: The History of Nuclear Research in Germany*, New York: Simon and Schuster (1967).

34 See p. 331, Cassidy, David C., *Beyond Uncertainty -- Heisenberg, Quantum Physics, and the Bomb*, Bellevue Literary Press (2009). See also p. 148, Powers, Thomas, *Heisenberg's War - the Secret History of the German Bomb* (Ref. [11]). Powers reports the same event, but provides additional information on pp. 515-516, Note 17. Powers, in particular, claims that Heisenberg wrote a letter to Goudsmit on October, 1948, in which he told him what he said. Powers gives the following reference: "Goudsmit papers, American Institute of Physics. The German Atomic Bomb, 120, quotes Heisenberg as saying 'as large as a pineapple.' Telschow told Armin Hermann the phrase used was 'about as big as an ananas' -- the German word for pineapple. Erich Bagge remembered that Heisenberg said 'about as big as a football.' [...]."

Heisenberg referred for the first time to "tons" in T1 after the internees were informed at 6:00 p.m. on August 6, 1945, that an atomic bomb was dropped on Hiroshima. During dinner that evening, Heisenberg made his disbelief abundantly clear by saying:[37] "*All I can suggest is that some dilettante in America who knows very little about it has bluffed them in saying: 'If you drop this it has the equivalent of 20,000 tons of high explosive' and in reality doesn't work at all,*" and "*I still don't believe a word about the bomb but I may be wrong. I consider it perfectly possible that they have about ten tons of enriched uranium, but not that they have ten tons*[38] *of pure* ^{235}U." (He later summed it up as follows: "*I am willing to believe that it is a high pressure bomb and I don't believe it has anything to do with uranium but that it is a chemical thing where they have enormously increased the speed of the reaction and enormously increased the whole explosion.*") Hahn's response is really remarkable:[39] "*But tell me why you used to tell me that one needed 50 kilograms of '235' in order to do anything. Now you say one needs two tons.*" Heisenberg's response must be seen as evasive: "*I wouldn't like to commit myself for the moment, but it is certainly a fact that the mean free paths are pretty big.*"

Heisenberg's disbelief can be explained by the generally acknowledged fact that he was convinced that Germany was ahead in nuclear research (a nuclear reactor that was "almost" finished), reinforced by Samuel Goudsmit, a friend of many years,[40] who, when interrogating Heisenberg on May 6, 1945,[41] answered Heisenberg's question about America's involvement in building a nuclear explosive by replying that[42] "*there had been more important things to do during the war,* and that *"there had been no efforts in that direction.*" Heisenberg's feeling of betrayal comes through loud and clear later that fateful Hiroshima day when, after the 9:00 p.m. BBC announcement, he became convinced that the bomb was atomic. Bernstein writes:[43] "*Heisenberg went on to complain bitterly that Goudsmit had lied to him very cleverly and thinks that he might at least have told him that their experiments in America were further advanced.*"

The findings in this paper and the fact that an estimate of 10-100 kilograms appeared in a German report in 1942, as mentioned above, demonstrate that Heisenberg must have been aware during the war that the critical mass is on the order of kilograms. We must conclude therefore that he did not reveal what he knew on the day of the bombing of Hiroshima, but instead chose to give an estimate he knew to be much too large.

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35 The New York Times published the obituary, "Manfred von Ardenne, 90, Dies; Was German Nuclear Physicist," on May 29, 1997. It noted that Baron von Ardenne (1907-1997), a prodigious inventor who held over 600 patents during his lifetime, was drafted by the Soviet Union after World War II. There, he demonstrated his ability to contribute to building nuclear explosives by developing a process for enriching uranium that played a key role in producing that country's first A-bomb. Ardenne, in his timeline at the end of his book (Ref. [1], p. 374) lists Heisenberg's visit on November 28, 1941, that of Hahn on December 12, 1941. He refers, however, to a time-frame of 1940 [a printing error] when he writes on p. 132: "*Bei Besuchen in Dahlem und Lichterfelde hatte ich 1940 sowohl Professor Otto Hahn als auch Professor Werner Heisenberg die Frage gestellt, wieviel Gramm des reinen Isotops Uran-235 zur Entfesselung einer momentan ablaufenden Kernkettenreaktion benötigt würden. Sie antworteten mir: 'Wenige Kilogramm.'* " Translation: "During visits in Dahlem and Lichterfelde in 1940 I asked Professor Otto Hahn as well as Professor Werner Heisenberg the question how many grams of pure Uran-235 would be required to trigger a chain-reaction. They answered: 'A few kilograms.' " Powers, in *Heisenberg's War* (Ref. [11], p. 450) wrote in Footnote 63, p. 577: "*This account was confirmed in detail by Ardenne personally in an interview at his laboratory in Dresden, May 17, 1989. He also showed me the guest book he began keeping when his laboratory opened in 1928; the unmistakable signatures of Heisenberg and Hahn were dated by them.*"

36 [3], p. 914.

37 [2], pp. 116-117.

38 [***Comment added***: The "ten tons" should be changed to "two tons," because that is what Hahn heard as quoted below.]

39 [2], p. 118.

From Heisenberg's emotional complaint that Goudsmit misled him, it nevertheless follows that he was eager to impress his colleagues during dinner with his disbelief that the bomb dropped on Hiroshima was atomic, an opinion solidified by his interrogation by Goudsmit to which we allude above. Heisenberg's bitterness about Goudsmit's misleading statement also implies that he would never have made this statement without Goudsmit's betrayal.

Because there is no simple explanation for his "tons" statement, we choose to consider as context his wish to use what he believed was the Germans' lead in nuclear research as a "bargaining chip" for concessions by the Allies -- a thought he most likely already had during the war. In this scenario, to be in the strongest position possible in the negotiating process Heisenberg envisioned, he must say nothing that might inform the Allies about the progress the Germans made during the war. That would only be revealed in the course of the negotiation he would initiate at the proper time. And we reasonably surmise that to protect his "bargaining chips," he "played it safe" by taking into account the possibility that the Allies were eavesdropping on what the internees said. And he camouflaged his knowledge by reverting back to the prevailing opinion about the critical mass before the war started as stated by Sir Charles Frank, in *The Farm Hall Transcripts*:[44] *At the outbreak of World War II, while it was common knowledge among scientists that the uranium isotope 235U was capable of producing a devastating explosion, no-one knew within wide limits what was the critical mass for 235U and many scientists would have made a guess in the order of tons.*"

The "bargaining chip" scenario is not as far-fetched as it might seem; others have speculated along the same lines. Bernstein mentions Heisenberg's assumption of German "superiority" as well as his fantasy about a bargaining chip when he writes in *Hitler's Uranium Club*:[45] "Heisenberg was still convinced -- and indeed remained convinced even after Hiroshima -- that his reactor experiments were significantly ahead of the Allies -- something that could eventually be used as bargain-

40 [11], pp. 7-12: In *Heisenberg's War*, Powers provides a view of the Heisenberg-Goudsmit relationship by way of a narrative about Heisenberg's friends' efforts during his lecture tour in 1939 to persuade him to leave Germany: "*The whole matter was hashed over again at great length in the last week of July at the University of Michigan in Ann Arbor, where Heisenberg stayed in the home of the Dutch physicist Samuel Goudsmit, a friend since 1925. [...] On his trip he had tried to explain himself one last time, and although he probably knew he had persuaded no one, he left thinking that his friends were still friends. With him on the Europa he brought a photograph of himself standing with one of them -- Samuel Goudsmit -- in front of Goudsmit's home in Ann Arbor. In Germany he would frame this photograph and place it on his desk. Six years later at war's end, after many removes, the photograph would still stand on his desk*."

41 The Alsos mission is described by Landsman's as follows ([10], Section Goudsmit): "Alsos was a scientific intelligence mission that followed the Allied troops in the wake of their invasion of Europe. Its initial purpose was to 'learn as soon as we could what the Germans might be able to do if they exerted every possible effort to produce an atomic weapon. After it had become clear that nothing was to be feared, its goal became to keep whatever scientists and scientific equipment that would be of any military value out of the hands of the Russian (and French) troops. Reporting directly to Leslie Groves in the US, its military commander in Europe was Boris Pash, and its scientific head was Sam Goudsmit."

42 [8], p. 108. See also Elisabeth Heisenberg, in *Das Politische Leben eines Unpolitischen* (p. 133), R. Piper & Co. Verlag (1980). She wrote: "Goudsmit sagte dazu lächelnd, man hätte Wichtigeres im Kriege zu tun gehabt und keine Anstrengungen in dieser Richtung unternommen."

43 [2], p. 131.

44 Frank, Sir Charles, *Operation Epsilon: The Farm Hall Transcripts*, Institute of Physics Publishing, Bristol and Philadelphia (1993), p. 4.

45 [2], p. 49.

ing chip with the Allies." Landsman, in his "Essay Overview: Getting Even with Heisenberg,"[46] also mentioned the idea of a bargaining chip: "[I]t seems that Heisenberg mainly wanted to complete a nuclear reactor in order to impress the Allies in peacetime, thereby hoping to secure both Germany's physics and his own leading role in it."[47]

On the subject of the internees' view of the possibility that the British were eavesdropping on the Germans, Elisabeth Heisenberg wrote[48] about the fact that some actually searched for listening devices, while others did not even care, and went on to say:[49] "*Heisenberg told me about it later when we met again. He said he had - jokingly [in his conversation with Diebner quoted below] - intended to get one over on the Americans or also the English and had said, in case they were listening: 'Really, one should not assume that in 'good old England' such Gestapo methods are usable.'* "

Yet Sir Charles Frank concluded[50] from the evidence of the tapes from Farm Hall that the German internees made no effort to take advantage of such a situation by "feeding" information to the Allies.

The topic of eavesdropping came up on July 6, 1945, when Heisenberg and Diebner had the following conversation at Farm Hall:[51]

Diebner:
I wonder whether there are microphones installed here?

Heisenberg:
Microphones installed? (laughing) Oh no, they're not as cute as all that. I don't think they know the real Gestapo methods; they're a bit old fashioned in that respect.

This seems to contradict our explanation of Heisenberg's "tons" remarks that assumes that Heisenberg took into account the possibility that the British were listening to what he said. But because the information Heisenberg wanted to protect could, in his view, determine what kind of research the Allies would allow the Germans to do, we nevertheless think that we are justified in surmising that Heisenberg's remarks were dictated by his concern that electronic surveillance might be taking place -- a precautionary strategy in a "high-stakes poker game." His remark about "*making money,*" quoted in the next paragraph, gives more credence to our bargaining chip scenario.

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46 Ref. [10].

47 [***Comment added***: Landsman references Hans Bethe's paper "Heisenberg & the German Bomb Mystery, The German Uranium Project," Physics Today (July 2000).

48 German original in Elisabeth Heisenberg's *Das Politische Leben eines Unpolitischen* (p.139), R. Piper & Co. Verlag (1980): "*Die Deutschen, an diese Möglichkeit von den Nazis gewöhnt, spekulierten über eine solche Möglixhkeit. Einige machten sich daran, hinter den Bildern und unter dem Teppich nach den Wanzen zu suchen. Andere reagierten mit mehr Gleichmut*."

49 German original in Elisabeth Heisenberg's *Das Politische Leben eines Unpolitischen* (p.139), R. Piper & Co. Verlag (1980): "*Heisenberg erzählte mir später, als wir uns wiedersahen, davon und sagte, er habe im Scherz -- den Amerikanern oder auch Engländern eins auswischen wollen und habe -- im Hinblick auf die Möglichkeit, daß sie es hörten -- gesagt: 'Man sollte doch eigentlich nicht annehmen, daß man sich in dem guten altem England solcher Gestapo-Methoden bediene!'* "

50 Frank, Sir Charles, *Operation Epsilon: The Farm Hall Transcripts*, Institute of Physics Publishing, Bristol and Philadelphia (1993), p. 3.

51 [2], p. 78.

The most surprising thing is probably that Heisenberg, even after it became clear to him on August 7, 1945, that a uranium atomic bomb was dropped on Hiroshima, thought that he could offer something worthwhile to the Allies; and we quote:[52] "*[After] the guests all heard Sir John Anderson speak [at 6 o'clock]*" Heisenberg said "*[i]f the Americans had not gotten so far with the engine as we did -- that's what it looks like -- then we are in luck. There is a possibility of making money.*"[53] This implies that Heisenberg, still at that time unaware of the plutonium bomb that would be dropped on Nagasaki a few days later, thought that the Allies were only manufacturing ^{235}U, neglecting development of a nuclear reactor.

Heisenberg's second "tons" remark took place in T2, when he knew that the Hiroshima bomb was atomic, in his private conversation with Hahn after dinner:[54]

Hahn:
How does the bomb explode?

Heisenberg:
In the case of a bomb it can only be done with the very fast neutrons. The fast neutrons in "235" immediately produce other neutrons so that the very fast neutrons which have a speed of -- say --1/30th that of light make the whole reaction. Then of course the reaction takes place much quicker so that in practice one can release these great energies. In ordinary uranium a fast neutron nearly always hits "238" and then gives no fission.

Hahn:
I see, whereas fast ones in the "235" do the same as the "238", but 130 times more.

Heisenberg:
Yes, if I get below 600,000 volts I can't do any more fission the "238," but I can always split the 235 no matter what happens. If I have pure "235" each neutron will immediately beget two children and then there must be a chain reaction which goes very quickly. Then you can reckon as follows. One neutron always makes two others in pure "235."That is to say, in order to make 10^{24} neutrons I need 80 reactions, one after the other. Therefore I need 80 collisions and the mean free path is about 6 centimeters. In order to make 80 collisions, I must have a lump of a radius of about 54 centimeters and that would be about a ton.[55]

The model Heisenberg used can be associated with the so-called random walk, and the result of 54 centimeters he obtained for the radius of the uranium sphere can be obtained from the following random walk formula for the critical radius: $\text{crit. radius} = (\text{length of step})\sqrt{\text{number of steps}}$, where the length of the step is equal to the mean free path Heisenberg mentioned, six centimeters, and the number of steps equals 80.

52 [2], p. 139. Bernstein, in Annotation 154, writes: "John Anderson, trained as a scientist, was Churchill's Chancellor of the Exchequer. He was the cabinet official in charge of the British nuclear program.

53 Note that this statement supports our bargaining chip scenario.

54 [2], pp. 128-129.

55 As Bernstein pointed out (Ref. [2], p. 130, note 103), Heisenberg actually made a mistake; the amount is 13 instead of two tons.

In coming up with a plausible scenario for Heisenberg's blunder, one must take into account that up to Hahn's question how an A-Bomb explodes, the discussions among the internees, after the earthshaking BBC announcement of the Hiroshima news at 9:00 p.m., proceeded in a way that did not show any emotional distress.[56] Consequently, it could not have affected Heisenberg's reasoning ability here when he discussed technical topics with Hahn.

The only reasonable explanation seems to us to be that Heisenberg needed a simple way to explain to Hahn -- a chemist, not familiar with nuclear physics -- how an atomic bomb explodes. Thus, the RW model, although "technically inaccurate," perfectly satisfied the requirement for simplicity.

The "fly in the ointment" here is the "tons" requirement derived from the RW model conflicts with the reality that an atomic bomb had been built. Because this contradiction is so glaringly obvious, it strengthens our argument that Heisenberg must have realized this and employed RW only for pedagogical reasons. That Hahn did not question it may be due to the fact that he was confused by Heisenberg's "tons" statement earlier that day which contradicted his recollection.

Comments on Heisenberg's Nuclear Project in WW II

Our analysis of Heisenberg's G39 report to German Army Ordnance[57] shows that he developed the theory only for the development of a nuclear reactor, laying no theoretical foundation for developing a nuclear explosive. Furthermore, it should be emphasized that Heisenberg mentioned nuclear explosives in his 1939/40 report on only two (out of a total of 41) pages, without mathematical reasoning.[58]

To extend his report to include that solution, he would have needed to use only well-established classical mathematics that he had learned from his attendance at lectures given by Professor Sommerfeld, his scientific advisor, at the University of Munich. Directly applicable was Sommerfeld's main lecture in 1923 on partial differential equations in physics which also[59] "*was probably most useful to Heisenberg in his work on his thesis problem: solving the [extremely] complicated equations for the stability and turbulence of flowing fluids.*" We show by mathematical/physical methods that Heisenberg COULD easily have expanded his stationary (time-independent) solution of the diffusion equation to the non-stationary (time-dependent) case -- although there is no HARD evidence that he did.

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56 [2], pp. 120-129.

57 See "Part II: The Missing Non-Stationary Solution of the Diffusion Equation" and "Part III: Hidden Information in Heisenberg's 1939/40 Report" below.

58 Heisenberg's 1939/40 report is contained on pages 378-418 of Ref. [4]. Nuclear explosives were only mentioned on p. 389. stating: "This explosive transformation of uranium atoms can only take place in almost pure U_{92}^{235}, because neutrons will be absorbed at resonance points of U_{92}^{238} even if only small amounts of impurities of U_{92}^{238} are present," and the summary on p. 396 reads: "Enrichment of U_{92}^{235} is the only method to make the volume of the machine small in comparison to 1 cbm. Moreover, it is also the only method to produce explosives which exceed the explosive power of the strongest available ones by several powers of ten."

59 [7], p. 150.

After Bernstein quickly read an early draft hereof, he told us[60] that it had been clear to him "*for a very long time that [Heisenberg] COULD have calculated the critical mass but he didn't.*" Bernstein noted that someone in the Uranverein seemed to have done so because a more or less correct number appeared in the 1942 summary report to German Army Ordnance. Bernstein also commented that he had always been puzzled by Heisenberg's exchange with Hahn at Farm Hall because Hahn thought that Heisenberg had a smaller number during the war and Ardenne seemed also to think so; and he found it very difficult to understand why if Heisenberg had done it right once, he did it wrong when he spoke to Hahn.

We emphasize that our focus is strictly on Heisenberg's role, and his omission does not mean that the Germans did not work on research that might have contributed to the development of an atomic bomb. This was conducted in the form of isotope separation, an effort that could lead to reduce the size of a nuclear reactor via the enrichment of uranium 235 as well as an effort to develop a nuclear explosive via the discovery of a way to obtain (almost) pure ^{235}U.

Bernstein states that[61] "*[in] February of 1943, [Harteck] and a colleague, Johannes Jensen, proposed using a double centrifuge for separating isotopes. [...] Harteck estimated that about 100 of these centrifuges would be enough to enhance the percentage of ^{235}U so that a reactor would operate with substantial less uranium. The so-called 'ultracentrifuge' was one of the enduring technological innovations to come out of the German project.*"

Committing the resources necessary for initiating what would undoubtedly have to become a very massive project for the actual construction of an A-bomb surely required evidence that such a project might succeed. The Allies had it in the form of the Frisch-Peierls[62] memorandum in March of 1940,[63] but, due essentially to the absence of a theory of nuclear explosives in Heisenberg's 1939/40 report, the Germans did not. This must have affected the way German physicists conducted their research, focusing on obtaining ^{235}U by methods already in use rather than on a more vigorous search for a method that could lead to the accumulation of enough enriched fuel to build A-bombs.

A final word on this topic: In contrast to Heisenberg's work on his G39 report, he revealed all he knew in the 1941/1942 time frame when a critical mass estimate of 10-100 kilograms -- most likely, as concluded by Walker,[64] with Heisenberg's involvement -- appeared in an anonymous report. Furthermore, Heisenberg mentioned the possibility of a plutonium bomb in the Harnack Haus meeting in Berlin on June 4, 1942, in his presentation:[65] "*At this point I would like to mention that due to the positive results up to now it cannot be ruled out that following the production of a uranium reactor it*

60 Private communication on October 1, 2009, with Prof. Jeremy Bernstein.

61 [2], pp. 40-41.

62 Rudolf Peierls (1907-1995) -- Physicist, educator, and author -- was born in Berlin, Germany; immigrated to England in 1933, and became naturalized British subject in 1940. A prominent physicist, who was involved to develop the atomic bomb during World War II, he is credited, along with Otto Frisch, with creating the formula which proved that only a small amount of uranium was needed to create an effective atomic bomb.

63 [14], p. 80. See their March of 1940 report titled "Memorandum on the Properties of a Radioactive 'Super Bomb.' "

64 [16], p. 216

65 [9], p. 88. German original: "Ich möchte an dieser Stelle erwähnen, dass es nach den bisherigen positiven Ergebnissen nicht ausgeschlossen erscheint, dass man nach Herstellung des Uranbrenners auf einem von v. Weizsäcker angegebenen Weg auch eines Tages zu Explosivstoffen kommen kann, die alle bisherigen um das Millionenfache an Wirksamkeit übertreffen.[...]."

might be possible one day via a route v. Weizsäcker outlined to come to an explosive that will exceed all presently existing ones in effectiveness millions of times." He alluded to the fact that a plutonium bomb required a nuclear reactor by saying:[66] "*The time needed for the technical development of such a reactor depends at the present time mostly on the availability of material, in particular on the production of heavy water. Besides these, however, a lot of scientific development work still has to be done. Taking into account the hurdles to overcome in such research it should also be recognized that new technological territories of highest importance can be discovered within the next few years.*" Furthermore, in his presentation in the same meeting he said:[67] "*Since we know that in America these problems are being worked on by engaging a large number of the best laboratories, Germany can ill afford not to proceed to address these questions. Even recognizing that such research normally takes a long time, one must take into account the possibility that in a long war with America, lasting several more years, the technical application of atomic energy might one day play a decisive role in the outcome of the war.*"

This meeting on June 4 was an important one, because Albert Speer's[68] presence made it possible to confirm or to overrule an earlier decision that the German army would relinquish its control of the nuclear project. That decision was allowed to stand, but nuclear research in the form of construction of a nuclear reactor was allowed to continue with low priority.

There is no conflict between these revelations and the claim that Heisenberg did not do his best to aid the German war effort in the spring of 1940 when he did not develop a theory of nuclear explosives. It can reasonably be inferred that in 1942 he revealed all he knew because he must have taken into account that someone else might sooner or later be able to calculate the critical mass, or German intelligence might have obtained such information -- putting his competence into question or, worse, opening himself to a charge of disloyalty. Although the Germans determined in 1942 that A-bombs could not be completed during the war, such an assessment would probably not been made two years earlier when the G39 report was finished in December of 1939.

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66 [9], p. 88. German original: "Die Zeit bis zur technischen Entwicklung eines solchen Brenners wird im Augenblick weitgehend von Materialbeschaffungsfragen bestimmt, inbesondere durch die Produktion des schweren Wassers. Aber abgesehen von den Materialfragen, muss noch viel wissenschaftliche Entwicklungsarbeit geleistet werden. Selbst wenn man die Schwierigkeiten einer solchen Entwicklungsarbeit in Rechnung setzt, wird man aber darauf gefasst sein müssen, dass hier in den nächsten Jahren ein Neuland von größter Bedeutung für die Technik erschlossen werden kann. [...]."

67 [9], p. 88. German original: "Da wir wissen, dass in Amerika an diesem Problem mit dem Einsatz einer großen Reihe der besten Laboratorien gearbeitet wird, kann man in Deutschland kaum auf die Verfolgung dieser Fragen verzichten. Selbst wenn man daran denkt, dass solche Entwicklungen meist lange Zeit brauchen, muss man dann, wennn der Krieg mit Amerika noch mehrere Jahre dauern sollte, mit der Möglichkeit rechnen, dass die technische Verwertung der Atomkernenergie eines Tages eine kriegsentscheidende Rolle spielen kann."

68 After the army had already decided to relinquish the project, Albert Speer, in his position as the "Reichsminister of Armaments and Munitions," became involved in the process of deciding whether or not to build an atomic bomb -- he could even overrule German Army Ordnance's decision. Powers, in *Heisenberg's War* (Ref. [11], pp. 142-146), gives a fascinating account of the rise of Speer to become, perhaps, the most influential member of the German government other than Hitler himself.

Part II: The Missing Non-Stationary Solution of the Diffusion Equation

In developing a theory for a nuclear explosive, the Allies started with the well established diffusion equation as discussed in *The Los Alamos Primer*,[69] a book that was used as a training manual by the Allies. This equation mathematically shows how the neutron density ($N(\text{space},t)$, neutrons per cm^3/sec) builds up as a function of space and time:[70]

$$\frac{\partial}{\partial t}N(\text{space},t) = D\nabla^2 N(\text{space},t) + \frac{(\nu - 1)}{\tau}N(\text{space},t) \tag{4}$$

Here, ν represents the average number of neutrons produced per fission, defined neutron number in *The Los Alamos Primer*,[71] and τ is the average time interval between fissions. Heisenberg employs $v = \frac{(\nu - 1)}{\tau}$, where v stands for the characteristic reciprocal time. D denotes the diffusion coefficient which indicates -- for a non-uniform neutron density -- how rapidly neutrons spread out throughout space, here a uranium sphere of radius R, and ∇^2 is the so-called Laplace operator. Although Eq. (4) is the starting point for the Allied and German nuclear research, there is a crucial difference in the next step. The Allies, starting a crash project near the end of 1942 to build an atomic bomb, developed the spherically symmetric non-stationary (time-dependent) neutron distribution as shown in Eq. (5).[72] In contrast, Heisenberg investigated only the stationary (time-independent, steady state) solution, restricted by the condition $\frac{\partial}{\partial t}N(\text{space},t) = 0$. As described in "Part III: Hidden Information in Heisenberg's 1939/40 Report" below, he took this approach because in nuclear reactor design one is interested in the equilibrium situation, when energy is continuously being freed at a rate which is time-independent.

In the Allied solution of the diffusion equation, described by[73]

$$N(\text{space},t) = N_1(r)e^{\mu t}, \tag{5}$$

69 [14], p. 25.

70 Serber, in Ref. [14], p. 26, presents this equation in an equivalent form as follows: $\dot{N} = D\Delta N + \frac{(\nu - 1)}{\tau}N$. Here, the symbol Δ is used for the Laplacian, as was customary then. Heisenberg, who also recognized that the diffusion equation is the one to use in nuclear research (see Ref. [4], p. 388) employs $\frac{d\rho}{dt} = D\Delta\rho + v\rho$ in his G39 report, and $\dot{n} = D\nabla^2 n + vn$ in his Farm Hall lecture on August 14, 1945 (Ref. [4], p. 174).

71 [14], p. 25.

72 [14], pp.25-33.

73 Heisenberg, in Ref. [2], p. 174, takes a similar approach in his Farm Hall lecture by defining $n = n_0 e^{\mu t}$ and Serber, in Ref. [14], p. 26, presents this as $N = N(\text{x,y,z})e^{\nu' t/\tau}$, where ν' is called the effective neutron number.

the spatial distribution depends only on the distance r from the center of the uranium sphere, and $e^{\mu t}$ indicates that the neutrons increase exponentially, with μ designating the characteristic exponential factor. (Heisenberg, in his August 14, 1945, lecture at Farm Hall, took the same approach,[74] but not in his G39 report, as shown below.)

The boundary conditions to be satisfied are: continuous neutron density N_1 and neutron flow j at the boundary of the uranium sphere and the tamper, imposed by the American scientists[75] and by Heisenberg in his Farm Hall lecture[76] when investigating atomic bombs, and by Heisenberg in his G39 report, in which he developed a theory for a nuclear reactor.[77] The neutron flow, in ordinary diffusion theory, is proportional to the gradient of the neutron distribution, $j = -D \text{ grad } N_1$. With these assumptions, Eq. (4) yields now, in agreement with *The Los Alamos Primer*[78] and Heisenberg's Farm Hall presentation,[79] the equation

$$\nabla^2 N_1 + \frac{\nu - \mu}{D} N_1 = 0 \tag{6}$$

to be solved to obtain the neutron density in the active region.

For the tamper, the simplifying assumptions made in *The Los Alamos Primer*[80] and in Heisenberg's Farm Hall lecture[81] is that (1) there is neither absorption nor generation of neutrons, and (2) the operation is near critical conditions (for which $\mu = 0$). The second term in Eq. (6) can thus be neglected, resulting in

$$\nabla^2 N_1 = 0 \tag{7}$$

for the equation to be satisfied in the tamper region.

Solving Eqs. (6) and (7) for N_1 with the above stated boundary conditions yields, in agreement with *The Los Alamos Primer*[82] and Heisenberg's Farm Hall presentation,[83] the following results (μ and critical radius, R_c) which hold for a very large tamper and for identical diffusion coefficients in the active and tamper regions. (More about the diffusion length l is given below.)

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74 [2], p. 174.

75 [14], pp. 30-31. The second condition is explicitly stated, but the first one is only implied when deriving the equation for k appearing in the obtained neutron distribution $N_1 = \frac{\sin(kr)}{r}$

76 [2], p. 175.

77 [4], p. 390.

78 [14], p. 26. If ν is replaced by $(\nu - 1)/\tau$ and μ by (ν'/τ) .

79 [2], p. 175.

80 [14], pp. 30-31.

81 [2], pp. 175-176.

82 [14], pp. 26-27. If ν is replaced by $(\nu - 1)/\tau$ and μ by ν'/τ .

83 [2], p. 178. Taking care of some obvious typing errors reveals that Eqs. (8) and (9) are, indeed, derived.

$$\mu = v\left[1 - \left(\frac{R_c}{R}\right)^2\right] \quad (8)$$

$$R_c = \frac{\pi}{2}\sqrt{\frac{D}{v}} = \frac{\pi}{2}l \quad (9)$$

The quantity $\frac{D}{v}$, introduced by Edoardo Amaldi and Enrico Fermi according to Heisenberg, is called the square of the diffusion length $\frac{D}{v} = (\text{diffusion length})^2 = l^2$.[84] The Allies[85] as well as Heisenberg[86] in his lecture at Farm Hall defined the critical radius as that value at which the neutron distribution neither decreases nor increases, where $\mu = 0$.

Although the critical radius formula was obtained via the non-stationary solution of the diffusion equation, it can also be derived from the stationary solution, as Bernstein points out.[87] This is also the reason that it is possible to obtain the critical radius formula for an atomic bomb from the equations Heisenberg derived for a nuclear reactor in his G39 report (see the Section "Derivation of the Critical Mass Formula For an Atomic Bomb" below), something that Heisenberg surely would have been able to do. Bernstein also expressed his surprise at Heisenberg's omission when he wrote:[88] "*What is remarkable is that he did not take the next step. He did not study the case of pure U~235! and ask how much would be needed to make a fast fission bomb.*"

The following statement made by Heisenberg during his Farm Hall lecture,[89] referring to Eq. (8), explains how a chain reaction develops.

"*We have then the diffusion equation according to which the neutrons spread. [...] There will certainly be solutions in which the neutron density on the whole decreases exponentially, and others in which it increases exponentially. Indeed, the following clearly holds: If the uranium sphere is small, more neutrons travel outwards than are produced in the interior, and the neutron density diminishes exponentially. On the other hand, if the sphere is made enormously large, the flux of neutrons outwards is negligible compared with the multiplication inside, and then the density increases exponentially.*"

An accurate description of the inner workings of an atomic bomb can thus be developed from a solution of the non-stationary form of the diffusion equation, something Heisenberg did not include in his G39 report to German Army Ordnance. In other words, he did not develop a theory for the workings of an atomic bomb in the spring of 1940, although he could easily have done so, as we show in "Part III: Hidden Information in Heisenberg's 1939/40 Report" below. We also show, however, that later, in the 1941/42 time frame, he told German authorities everything that he knew about nuclear explosives, including his conclusion that the technical hurdles to be overcome in the production of

84 [4], p. 388.
85 [14], pp. 27-28.
86 [2], pp. 174-178.
87 [3], p. 914.
88 [3], p. 915.
89 [2], p. 174.

an A-bomb were so high that many years would be required for mastering them. We explain that this seemingly contradictory behavior does not present a contradiction in Section "Comments on Heisenberg's Nuclear Project in WW II" in "Part I: Heisenberg & The Critical Mass: Resolved" above.

The argument that developing a nuclear reactor fits into the category of building an atomic bomb due to the fact that the reactor could produce plutonium which then could lead to a plutonium atomic bomb does not apply here, because it was not known at the time Heisenberg issued the second part of his 1939/40 report to German Army Ordnance, in February of 1940.

Part III: Hidden Information in Heisenberg's 1939/40 Report

The Embedded Critical Mass Concept

To lay the groundwork for a nuclear reactor theory, Heisenberg modeled in his G39 report a reactor with a physical configuration consisting of a uranium sphere of radius R with a neutron source of strength n neutrons/second at its center and two alternative geometries, one with the uranium surrounded by air and another one with a tamper. The purpose of the neutron source in the center was primarily to get a chain reaction started. Heisenberg's key observation was that the phenomena -- diffusion, scattering, fission, and absorption -- which take place in a nuclear reactor can be described by the diffusion equation.

As one of the central results in his G39 report,[90] Heisenberg derived the spherically symmetric stationary (steady state) solution for the neutron density, $N_1(r)$, which depends only on the distance from the center of the sphere, r. To arrive at such a solution, one sets $\frac{\partial}{\partial t}N_1(r) = 0$, leading to

$$\nabla^2 N_1 + \frac{\nu}{D}N_1 = 0 \tag{10}$$

$$\nabla^2 N_1 = 0 \tag{11}$$

as the equations to be solved to obtain N_1 in the active and tamper region, respectively. Starting with these equations, Heisenberg obtained the appropriate neutron densities in the active and the tamper region. And from these he derived a formula for the number of neutrons, n' neutrons per second, escaping through its outer boundary in an equilibrium situation, ending up with the ratio $\frac{n'}{n}$, where n, as defined above, stands for the "number of neutrons generated by the neutron source per second." The result for a tamper design was:[91]

90 [4], pp. 388 & 390.
91 [4], p. 390, Eq. (45).

$$\frac{n'}{n} = \frac{1}{\cos\left(\frac{R}{l}\right)} \qquad (12)$$

This led Heisenberg to the important result that a trigger point, defined here as a critical point[92] (radius), R_c, exists at which n' approaches infinity and above which n' becomes negative -- meaning that a steady state solution does not exist for a radius R larger than the critical point (radius).[93] It also follows that the neutron source (n) at the center has no influence if R approaches the critical point (radius) and thus can be neglected -- in which case the reactor has reached criticality, running by itself. In summary, this trigger point, embodied in the following formula derived from Eq. (12), exactly reflects what was elsewhere defined as the critical radius.

$$R_c = \left(\frac{\pi}{2}\right) l \text{ (for natural or enriched U, for slow neutrons)} \qquad (13)$$

(The trigger point, critical radius, is reached when the cosine becomes zero in Eq. (12), when its argument becomes equal to $\frac{\pi}{2}$.) This result holds for a very large tamper and for identical diffusion coefficients in the active and tamper regions. Heisenberg's n'/n ratio is of practical significance. Since n is known from the design and n' can be measured, the ratio n'/n can be determined experimentally. Thus, experiments can demonstrate that neutron multiplication occurs whenever this ratio exceeds one. Also, knowing the radius R of the uranium sphere and measuring the n'/n ratio, one can determine the diffusion length experimentally from Eq. (12).

Heisenberg gives more details for a design without a tamper, a uranium sphere surrounded by air, for which $\frac{n'}{n} = \frac{\left(\frac{R}{l}\right)}{\sin\left(\frac{R}{l}\right)}$ holds.[94] For the case where the radius R is much smaller than the diffusion length l, he derived:[95]

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92 Heisenberg discussed in Ref. [4], p. 388 and pp. 389-390, what happens below and above a certain radius of the uranium sphere; but he did not designate a symbol nor a name for this quantity. In order to identify it, the name "critical point," also employed in introductory comments to the wartime uranium project (Ref. ([4], p. 366), was used in this paper.

93 [4], p. 389: Heisenberg discusses here a nuclear reactor that does not employ a tamper. (The same reasoning holds for a tamper design, if πl is replaced by $\left(\frac{\pi}{2}\right) l$.) We translate: "When R draws nearer the value πl, n' approaches infinity. When $R > \pi l$, a stationary neutron distribution does not exist anymore where the concentration is positive throughout, due to $\frac{\partial N}{\partial t} > 0$ according to (30) [Eq. (4) in this paper] an unlimited neutron increase takes place." See also Eq. (12). Because it turns out that the critical radius formula obtained for an atomic bomb (Eq. (9)) and the one for the critical point here have the same form, we took the liberty of using the same symbol, R_c, here.

94 [4]. p. 389, Eq. (36).

$$\frac{n'}{n} = 1 + \left(\frac{1}{6}\right)\left(\frac{R}{l}\right)^2 \tag{14}$$

Because Heisenberg's stationary (time-independent) solution dictates that there cannot be a build-up of neutrons in the active region, the number of generated net fission neutrons must be in balance with the neutrons escaping at the boundary of the uranium sphere, and at ($R = R_c$) the reactor achieves criticality as discussed above. This, however, defines the critical mass, i.e., as stated in *The Los Alamos Primer*:[96] "*At the critical radius the net number of neutrons produced by fission just equals the number escaping across the surface of the sphere per fission.*"

Derivation of the Critical Mass Formula For an Atomic Bomb

Why did Heisenberg limit himself to nuclear reactor theory? This is surprising because the same physical configuration and mathematical concepts he used for analyzing a nuclear reactor, a uranium sphere surrounded by air or a tamper, also apply to a nuclear explosive. Although a nuclear reactor requires a moderator in which fast fission neutrons are slowed down, it is not accounted for in the physical model; its influence is taken into account by employing slow neutrons in the active region of a nuclear reactor. Solving the diffusion equation for the case where the nuclear reactor has reached criticality (not requiring a neutron source at the center) we end up with the same physical model for both, a nuclear reactor and a nuclear explosive.

Technically one can fill the gap from Heisenberg's observations in his G39 report to the concept of critical mass for an atomic bomb, as used by the Allies, easily: In obtaining a solution, absorption does not have to be considered in either case. Absorption by ^{238}U can be neglected in a nuclear reactor by assuming the presence of slow neutrons (generated in a moderator by slowing down fast fission neutrons); it can also be neglected in a nuclear explosive by using almost pure ^{235}U since, by definition, the ^{238}U content is reduced to a level were absorption can be ignored. Heisenberg's requirement that almost pure ^{235}U is needed in a nuclear explosive[97] implies that fast neutrons are involved in fissioning this active material. (Thus it is not necessary to slow down fast fission neutrons, mandatory in nuclear reactors, to avoid absorption by ^{238}U.)

95 [4], p. 389, Eq. (37). Note that this equation is of the form $\frac{x}{\sin(x)}$ which, for small values of x, can be expressed as $x\left(x - \frac{x^3}{6}\right)^{-1} \approx \left(1 - \frac{x^2}{6}\right)^{-1}$. This in turn, for small values of x, becomes $1 + \frac{x^2}{6}$.

96 [14], p. 28.

97 [4], pp. 389 and 396. See also the paragraph before and after the indented two paragraphs in Section "The Flaws in Rose's Technical Findings" above. The first indented paragraph reads: This explosive transformation of uranium atoms can only take place in almost pure U_{92}^{235}, because neutrons will be absorbed at resonance points of U_{92}^{238} even if only small amounts of impurities of U_{92}^{238} are present.

Furthermore, the same mathematical concept based on the diffusion equation Heisenberg had developed for a nuclear reactor also would be essential for a theory of a nuclear explosive. In both situations, nuclear reactor and nuclear explosive, the physical phenomena to be accounted for are the same, i.e., diffusion, fission, scattering, and the effect of excess fission neutrons. Because Heisenberg, in his work on a nuclear reactor, employed the same boundary conditions stated above for a nuclear explosive, it follows now that his result for the critical radius shown in Eq. (13) for a nuclear reactor also applies for a nuclear explosive.

In other words, these facts make it possible to deduce the critical radius formula for an atomic bomb from the one for a nuclear reactor, Eq. (13), by appropriately interpreting its parameters, i.e., the diffusion length is now associated with almost pure ^{235}U and fast neutrons. For a tamper design, this reasoning leads to the following critical radius formula for an atomic bomb:

$$R_c = \left(\frac{\pi}{2}\right) l(\text{for pure, or almost pure, } ^{235}U\text{, for fast neutrons}) \qquad (15)$$

Heisenberg did not obtain a detailed formula for the diffusion length, in terms of parameters that could be measured. There was no need to do so, because the diffusion length could be measured for the available material used in a nuclear reactor, natural uranium. This was not possible, however, for bomb fuel, almost pure ^{235}U, because the Germans never obtained it. Consequently, for estimation purposes, a detailed equation for the diffusion length is required to obtain the critical radius (mass) formula for a nuclear explosive. This can be derived by using only the definition and quantities defined in Heisenberg's G39 report as shown below. (To show that the same information can be found in Heisenberg's Farm Hall lecture, we also provide the appropriate references associated with this lecture.) And we start with the definition of diffusion length, $l = \sqrt{\frac{D}{v}}$,[98] and derive from it the more detailed formula $l = \sqrt{\frac{\lambda_s \lambda_f}{3(\nu - 1)}}$,[99] where λ is the average distance a neutron travels between events (mean free path, MFP), λ_s for the average length between scattering and λ_f

98 A discussed above, the square of the diffusion length is defined $\frac{D}{v} = (\text{diffusion length})^2 = l^2$. This is mentioned in Ref. [4], p. 388 as well as in Ref. [2], p. 174.

99 It is interesting to observe that Heisenberg, in his G39 report (Ref. [4], p. 385), derived a similar formula for the diffusion length associated with absorption in the tamper region of a nuclear reactor: $l = \sqrt{\frac{\lambda_r \lambda_{th}}{3}}$, where λ_r designates the mean free path (MFP) for absorption and λ_{th} the MFP for scattering of thermal neutrons. The term $(\nu - 1)$ does not appear, because it is assumed that no neutrons are generated due to fission.

between fissioning. We arrive at this result by employing the expression for the diffusion coefficient[100] $D = \frac{\lambda_s \upsilon}{3}$, where υ identifies the velocity of the neutron, i.e., $\upsilon = \frac{\lambda_f}{\tau}$, and using (as explained above) $\nu = \frac{(\nu - 1)}{\tau}$ for the characteristic reciprocal time. The critical radius can now be expressed as follows:

$$R_c = \left(\frac{\pi}{2}\right) l = \left(\frac{\pi}{2}\right) \sqrt{\frac{\lambda_s \lambda_f}{3(\nu - 1)}}.^{101} \tag{16}$$

Critical Mass Estimation

When obtaining the detailed equation (16) for the critical radius, starting with $R_c = \left(\frac{\pi}{2}\right) l$ (Eq. (15)), we show above that this could be done with information contained in Heisenberg's G39 report. We show next that an estimate, although a crude one along the lines Frisch and Peierls pursued, can also be obtained. (Frisch and Peierls demonstrated, in their March of 1940 report titled "Memorandum on the Properties of a Radioactive 'Super Bomb,' " that building an atomic bomb might be feasible.)

To get a feeling for the magnitude, we assume (as Frisch and Peierls did) that each scattering results in fission, equivalent to setting scattering MFP = fission MFP, $\lambda_s = \lambda_f$. And we may use λ_s = 3.4 centimeters since, according to Heisenberg, the cross section is equal to 6×10^{-24} square centimeters[102] (corresponding to[103] MFP = 20.4/6 = 3.4 centimeters for uranium) for a wide range of materials. Using this value, together with a worst case value of two for the neutron number in Eq. (16) yields an estimate of $\left(\frac{\pi}{2}\right) \sqrt{\frac{1}{3}}\, \lambda_s = 3.08$ centimeters for the critical radius of an atomic bomb, resulting in a critical mass of 2.3 kilograms. This admittedly overly optimistic result nevertheless indicated that further research into nuclear explosives should be pursued.

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100 [4], p. 385 as well as Ref. [2], p. 173.

101 The same relation, derived by assuming that scattering is random, was used in *The Los Alamos Primer*, but then a correction factor was calculated for the "scattering MFP" to take non-randomness into account. (Ref. [14], pp. 73-74.) This change was mathematically dealt with by replacing "scattering MFP, λ_s," with "transport MFP, λ_t."

102 [2], p.170. Heisenberg said in his Farm Hall lecture: "For scattering again, we know that for all heavy elements the cross section is about 6 at high energies. In uranium we know it pretty exactly, for it is, I think equal to 6.2×10^{-24} in '238.' In lead it is slightly less, but they are all in this region. So we can say that for scattering the cross section is pretty certainly in the neighborhood of 6×10^{-24} and so the MFP is about 3.7 centimeters." To calculate the mean free path (MFP) Heisenberg used the formula 22/6 ≈ 3.7; he should have used, as Bernstein pointed out (Ref. [2], p. 170, Annotation 119), 20.4/6 = 3.4 cm. See also Footnote 103.

103 This formula was given in Heisenberg's G39 report (Ref. [4], p. 385) in its general form as follows: MFP = [1/(number of uranium nuclei per cubic centimeters)(cross section)]. Employing 4.902×10^{22} for the number of nuclei per cubic centimeters in uranium yields MFP = 20.4/(cross section), if the cross section is given in units of 10^{-24} square centimeters. Heisenberg called this relation a "*handy formula*" (Ref. [2], p. 170), but instead of using 20.4 he wrongly employed 22 for the constant in his Farm Hall lecture as Bernstein pointed out (Ref. [2], p. 170, Annotation 119). A similar approach can be used to get the corresponding value for plutonium.

Heisenberg’s Unexplained Critical Mass Formula at Farm Hall

In his lecture at Farm Hall on August 14 1945, Heisenberg started out with the following equation for the critical radius of an atomic bomb:[104] $R_c = \left(\frac{\pi}{2}\right) l$. But, surprisingly, instead of defining the diffusion length $\frac{D}{v} = (\text{diffusion length})^2 = l^2$ and derive from it the more detailed formula

$$l = \sqrt{\frac{\lambda_s \lambda_f}{3(\nu - 1)}}, \tag{17}$$

Heisenberg stated in his lecture on August 14, 1945:

$$\text{diffusion length } l = \frac{q}{\sqrt{\text{fission cross section}}}, \tag{18}$$

where q is a constant. He introduced this constant (q = 6.2 square centimeters) without explanation in an expression for the diffusion length (l) by stating:[105] “*Since the date of Fermi's work this quantity* $\frac{D}{v}$ *has been called the square of the diffusion length* l^2*.”*[106] *[...] This l comes out as 6.2 {square] centimeters*[107] *divided by the square root of the fission cross section, expressed in units of* 10^{-24} *[square centimeters]. If the fission cross section is just* $1x10^{-24}$*, the diffusion length is 6.2 centimeters.*” Heisenberg relates the diffusion length to the critical radius, R_c, by stating:[108] “.... *where* R_c *is a critical length,* $R_c = (\pi/2)\ l$*, which is approximately equal to 9.7* $cm^2 / (\text{fission cross section})^{1/2}$*.”*[109]

Loosely speaking, the fission cross section relates to the probability that fission will occur. Cross section is explained in Bernstein's *Hitler's Uranium Club*, as follows:[110]

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104 Frank, Sir Charles, *Operation Epsilon: The Farm Hall Transcripts*, Institute of Physics Publishing, Bristol and Philadelphia (1993), p. 133. This reference is quoted instead of Bernstein’s *Hitler's Uranium Club - The Secret Recordings at Farm Hall* (Ref. [2]) due to typing errors occurring in the English (p.178) as well at the German version (p. 199). Note that Heisenberg’s R_k was replaced with R_c for editorial reasons.

105 [2], p. 174.

106 [***Comment added***: Note that D stands for the diffusion coefficient (in square centimeters/second) and v for the characteristic reciprocal time (in 1/second). $\frac{D}{v}$ therefore has the dimension square centimeter.

107 [***Comment added***: It is obvious that the dimension should be square centimeters instead of centimeters. With l and $\sqrt{\frac{D}{v}}$ in centimeters, the constant q (= 6.2 in this case) must have the dimension square centimeters.]

108 Frank, Sir Charles, *Operation Epsilon: The Farm Hall Transcripts*, Institute of Physics Publishing, Bristol and Philadelphia (1993), p. 133.

109 Instead of using the fission cross section, he could have employed the fission MFP via the relation MFP = 20.4/cross section (Footnote 103). One could argue that the distance traveled by neutrons between events in nuclear processes would intuitively be better understood than the cross section, although that parameter can be measured, whereas MFP is derived by using the above equation.

110 [2], pp. 18-19.

"Physicists introduce the term cross section as a measure of probabilities for fissioning a nucleus or indeed any other nuclear process involving collisions. A cross section has the dimension of an area. Peierls has given a nice illustration. Suppose a window is constructed out of glass that is sufficiently strong that it breaks only one in ten times when a boy throws a baseball at it. The effective area -- or cross section -- for breaking the window is then a tenth of its actual geometric area. The factor of a tenth gives us a measure of the strength of the glass. Nuclear cross sections are tiny by macroscopic standards, but large by subatomic standards. In centimeters, a typical nuclear cross section is something like 10^{-24} square centimeters [...] this was such a big number, as compared, say, to the size of an electron, they called it a 'barn' -- as big as a barn door."

> By assuming now[111] a range for the fission cross section of 0.5 to 2.5 (in units of 10^{-24} square centimeters), Heisenberg arrived at a range of 6.2 to 13.7 centimeters for the critical radius. Note that for $2.5\text{x}10^{-24}$ square centimeters, one obtains $R_c = 9.7/(2.5)^{1/2} \approx 6.2$ centimeters, and $0.5\text{x}10^{-24}$ square centimeters yields $R_c = 9.7/(0.5)^{1/2} \approx 13.7$ centimeters, corresponding to a critical mass of 19 to 205 kilograms.[112]

It is surprising that Heisenberg did not use the critical radius equation he developed in his Farm Hall lecture, identical to the one we derived from Heisenberg's G39 report to German Army Ordnance, Eq. (16)), to obtain a numerical value for the critical mass. Let us first establish the numerical values of the parameters Heisenberg used in his Farm Hall lecture, starting with his statement relating to the neutron number by quoting from Bernstein's *Hitler's Uranium Club*:[113] "*Then there is an important quantity we need, the multiplication factor [Vermehrungsfaktor*[114]*]; i.e., the number of neutrons produced from a collision which results in fission of 235. Since we know the multiplication factor for thermal neutrons from our Berlin experiments, we calculate not per fission, but per thermal neutron absorbed. Now for thermals, the cross section for fission is about 3 and that for absorption about 6.2, i.e., we actually get only half the true multiplication factor if we take our figures. [...] Our figure is 1.18, so we can say the multiplication factor is really 1.18 x 2 or, roughly speaking, between 2 and 2.5.*"

Furthermore, Heisenberg stated at the start of his Farm Hall lecture:[115] "*In the fission, some neutrons are produced. These neutrons behave like the first, and so the chain reaction goes on. If one had an indefinitely large amount of ^{235}U, the chain reaction could go on indefinitely, for two to three neutrons would always result from each one of fission. These two or three would repeat the process, and so it would go on. Thus the total number of neutrons would increase exponentially. The multiplication of neutrons is however in competition with the process by which neutrons escape from the mass.*[116] *If in*

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111 [2], p. 178.

112 With a density of 19 grams per cubic centimeters for uranium, the critical mass (in grams) can be expressed as critical mass $= 19(4\pi/3)$ x (critical radius)^3 = 79.587 x (critical radius)^3 . For a range of 6.2 to 13.7 centimeters for the critical radius one obtains therefore a corresponding critical mass of 18.96, about 19, to 204.65, about 205 kilograms. Heisenberg must have used a different value for the uranium density because he came up with 16 kilograms. (See also Bernstein's comment on p. 178, annotation 208, Ref. [2].)

113 [2], p. 171.

114 [***Comment added:*** This technical term, occurring in the German version of the transcript, was translated "multiplication factor."]

115 [2], p. 169-170.

fact one has a finite mass of uranium, those neutrons at the surface which are moving outwards escape, and have no chance of taking part in fissions. So the question arises whether this loss of neutrons by escape from the mass, is greater or less than the gain of neutrons arising from the production inside. To calculate this, or convince oneself, it is necessary to have the cross sections and mean free paths."

From the above statements, (1) "*so we can say the multiplication factor is [...] roughly speaking, between 2 and 2.5*," and (2) "*[i]f one had an indefinitely large amount of U235, the chain reaction could go on indefinitely, for two to three neutrons would always result from each one of fission*," it can be inferred that the multiplication factor can be interpreted as the net number of neutrons produced per fission, i.e., equal to the "neutron number minus one."[117]

Although the Vermehrungsfaktor (multiplication factor) Heisenberg introduced in his Farm Hall lecture with[118] "*Dann ist eine wichtige Groesse, die man noch braucht, der Vermehrungsfaktor, also die Anzahl der Neutronen, die bei einem Stoss, bei einer Spaltung von 235 herauskommen*" ("*Then there is an important quantity we need, the multiplication factor; i.e., the number of neutrons produced from a collision which results in fission of 235*"[119]) is a parameter that describes what happens at an atomic scale, Heisenberg used the same word, Vermehrungsfaktor, macroscopically. That is, he wrote that the Vermehrungsfaktor[120] "*[ist] das Verhältnis der austretenden Neutronen zu den eintretenden*" ([is] the ratio of the escaping neutrons to that of the entering) when he made comments in conjunction with the experiment[121] that achieved Germany's first actual neutron multiplication due to fission sometime late in 1941 -- a modest but real increase of 13% measured at the outer wall. Here, he determined the Vermehrungsfactor that he mentioned by dividing the measured number of generated neutrons by the known number of neutrons produced by the neutron source.

Looking at the generic meaning of the Vermehrungsfaktor in this macroscopic situation, "number of generated neutrons divided by the known number of neutrons produced by the neutron source," one can deduce that it translates in the microscopic situation to "number of net neutrons generated per fission divided by the number of neutrons initiating fission." Thus it becomes in that case "number of net neutrons generated per fission" and this quantity, as argued above, is identical to "neutron number minus 1" employed by the Allies as well as used in this paper.

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116 [***Comment added:*** Bernstein remarks ([2], p. 170) in Annotation 118: "This is the first inkling in these reports that Heisenberg has begun the grasp the notion of a critical mass properly." Note that this is in contrast to our finding as discussed in the Section "The Embedded Critical Mass Concept" in "Part III: Hidden Information in Heisenberg's 1939/40 Report" above.]

117 Note that the neutron number designates the number of neutrons produced in a fission. Since the neutron triggering this event is absorbed in the process, the net number of neutrons produced is thus (neutron number - 1).

118 [2], p. 193.

119 [2], p. 171.

120 [4], p.546.

121 R. & K. Döpel and W. Heisenberg are the authors of the report describing the experiment which is listed in Ref. [4], *Werner Heisenberg's Gesammelte Werke / Collected Works, Series A/Part II, Original Scientific Papers / Wissenschaftliche Originalarbeiten,* pp. 536-544. The report titled "Der experimentelle Nachweis der effektiven Neutronenvermehrung in einem Kugel-Schichten-System aus D_2O und Uran-Metall" (The experimental Proof of the effective Neutron increase in a Sphere-Plate-System consisting of D_2O and Uranium Metal) is undated, but Döpel later gave the date of the experimental proof as April 1942 (Ref. [4], p. 536).

The G39 report lists measured scattering cross sections for natural uranium, 10 for slow and 6 for fast neutrons, in units of 10^{-24} square centimeters.[122] The corresponding scattering mean free paths are 2.0 and 3.4 centimeters, respectively, obtained via the equation mean free path = 20.4/fission cross section. A reasonable approach for getting an estimate for the scattering MFP in ^{235}U would therefore be to use the scattering MFP for natural uranium.[123] Since Heisenberg was aware of the fact that fast neutrons come into play in nuclear explosives,[124] 3.4 centimeters should be selected here. That this is a reasonable approach follows from Heisenberg's statement in his Farm Hall lecture:[125] "*For scattering again, we know that for all heavy elements the cross section is about 6 [in units of 10^{-24} square centimeters] at high energies.*" Although Heisenberg made this statement in 1945, it is reasonable to assume that he was aware of it when he wrote his report in the 1939/40 time-frame.

In his Farm Hall lecture, Heisenberg provided the following numerical values for relevant parameters to calculate the critical radius (mass): He estimated[126] (2-2.5) for the "multiplication factor" (neutron number minus one), 6.0 (in units of 10^{-24} square centimeters) for the scattering cross section, corresponding to a scattering MFP of 3.4 centimeters, and (0.5-2.5) for the fission cross section, also in units of 10^{-24} square centimeters, corresponding to 40.80 and 8.16 centimeters for the fission MFP, respectively. Using Eq. (16), derived from information contained in the G39 report, which could also have been obtained from Heisenberg Farm Hall lecture, with these parameters yields:[127]

$$\begin{aligned}\frac{\pi}{2}\left[\sqrt{\frac{(3.4)(8.16)}{3(2.5)}}\right] \leq \text{ critical radius} &\leq \frac{\pi}{2}\left[\sqrt{\frac{(3.4)(40.8)}{3(1)}}\right] \\ 3.02 \leq \text{ critical radius} &\leq 10.68 \quad \text{centimeters} \\ 2.20 \leq \text{ critical mass} &\leq 96.95 \quad \text{kilograms}\end{aligned} \tag{19}$$

We converted cross sections to mean free paths via the relation MFP = 20.4/cross section and used a neutron number of two, Vermehrungsfaktor of one, for a worst case design.

122 [4], p. 385, Table 2.

123 Since ^{235}U was never produced by the Germans during all the war years, the correct value for the scattering cross section of bomb material could not be obtained.

124 See the paragraph before and after the indented two paragraphs in Section "The Flaws in Rose's Technical Findings" in "Part I: Heisenberg & The Critical Mass: Resolved" above.

125 [2], p.170.

126 [2], pp. 172, 171, and 178, respectively. See also Frank's *Operation Epsilon: The Farm Hall Transcripts*,), pp. 127, 126 & 133, respectively.

127 The critical mass, in grams, is calculated with the aid of the following relation (see Footnote 112): critical mass = 79.587x(critical radius)^3. Based on the range of 3.02 to 10.68 centimeters for the critical radius, the critical mass can thus be calculated as follows: 79.587x3.02^3 = 2,192 grams, about 2.2 kilograms, to 79.587x10.68^3 = 96,952 grams, about 97 kilograms.

The discrepancy between this range of critical mass and Heisenberg's result of 19 to 205 kilograms is due to the unexplained factor of 6.2. The remedy is to use Heisenberg's short cut, expressed in Eq. (18), and come up with a factor (q), which can be derived by comparing Eq. (18) with Eq. (17), $l = \sqrt{\frac{\lambda_s \lambda_f}{3(\nu - 1)}}$, equivalent to $l = \sqrt{\frac{1}{3(\nu - 1)}(\lambda_s)^2\left(\frac{\lambda_f}{\lambda_s}\right)}$. Using the relation MFP = 20.4/cross section yields $l = \sqrt{\frac{1}{3(\nu - 1)}\left(\frac{20.4}{\sigma_s}\right)^2\left(\frac{\sigma_s}{\sigma_f}\right)}$, or $l = 20.4\sqrt{\frac{1}{3(\nu - 1)\sigma_s}} / \sqrt{\sigma_f}$, where σ_s and σ_s stand for the scattering and fission cross section, respectively. Comparing it with Eq. (18) yields

$$q = 20.4\sqrt{\frac{1}{3(\nu - 1)\sigma_s}}. \tag{20}$$

To obtain estimates for (q) in line with the assumptions made in the Farm Hall lecture, we must use a range of one (smallest acceptable value for the creation of a chain reaction) to Heisenberg's maximum estimated value of 2.5 for the number of net neutrons per fission, together with 6.0 in units of 10^{-24} square centimeters for the scattering cross section -- a value Heisenberg had high confidence in. Inserting these numerical values in Eq. (20) yields the following result:

$$\begin{aligned} 20.4)\sqrt{\frac{1}{3(2.5)6}} \leq\ & q \leq (20.4)\sqrt{\frac{1}{3(1)6}} \quad \text{square centimeters} \\ 3.041 \leq\ & q \leq 4.808 \quad \text{square centimeters} \end{aligned} \tag{21}$$

It follows that the acceptable range for q is 3.041 to 4.808 square centimeters. Choosing 4.81 for a worst case design to replace 6.2, which falls outside the tolerated range -- resulting in an upper bound for the critical radius -- yields: (Note that we use Heisenberg's estimated range of 0.5 to 2.5 for the fission cross section, in units of 10^{-24} square centimeters.)

$$\begin{aligned} 4.81/\sqrt{2.5} \leq\ & \text{diffusion length} \leq 4.81/\sqrt{0.5} && \text{centimeters} \\ 3.042 \leq\ & \text{diffusion length} \leq 6.802 && \text{centimeters} \\ \left(\frac{\pi}{2}\right)3.042 \leq\ & \text{critical radius} \leq \left(\frac{\pi}{2}\right)6.802 && \text{centimeters} \\ 4.78 \leq\ & \text{critical radius} \leq 10.68 && \text{centimeters} \\ 8.69 \leq\ & \text{critical mass} \leq 96.95 && \text{kilograms} \end{aligned} \tag{22}$$

Factoring out the fission cross section in the equations relating to the diffusion length, Eq. (18), has the advantage that[128] "*[t]his most difficult*" to estimate part is treated separately. It follows that q, see Eq. (20), contains the scattering parameter and neutron number, both easily estimated as demonstrated above in the discussion leading to Eq. (21). Why did Heisenberg not justify this approach by explaining how he arrived at the factor of 6.2? He practically performed a magician's

128 Remark by Heisenberg in his Farm Hall lecture (Ref. [2], p. 170).

hat trick, and it is surprising that none of his colleagues asked him how he developed this short-cut. The significance of Heisenberg's strategy for calculating the critical mass in his Farm Hall lecture is that it indicates that he may have used this approach before, making it likely that he employed the equation $\text{critical radius} = (\pi/2)(q/\sqrt{\text{fission cross section}})$ during the war -- perhaps as early as 1939, because all the information needed to develop this equation is contained in the first part of the report which he wrote in December of 1939. Because he also erred by using the factor 22 instead of 20.4 in his "Faust Formel" (handy formula),[129] as he called it,[130] MFP = 22/cross section, it is conceivable that he mistakenly employed 6.2 instead of 4.81 (or something close to it) for q.

Summary & Conclusions

We have shown above:

1. Heisenberg's nuclear reactor theory, developed in his G39 report to German Army Ordnance, demonstrates his awareness (1) of the critical mass concept, (2) that a tamper, surrounding the uranium sphere, cuts the critical radius theoretically in half, and (3) that a chain reaction in atomic bombs involves fast neutrons. The starting point of his research is the diffusion equation, an approach also employed by the Allies. We also showed that the critical radius formula as well as an estimate of kilograms for the critical mass can be derived from information contained in his report. This shows that the random walk model employed by Heisenberg in 1945 at Farm Hall, which led to a very large overestimate of the critical mass -- in tons -- that would have stopped any plan to develop an atomic bomb in its tracks, is a step backward from the sophisticated diffusion equation he used in 1939.

2. It appears from available evidence that Heisenberg did not use his skills to extend his investigation of nuclear reactors to nuclear explosives. This applies in particular to extension of his diffusion equation to a region with fast neutrons in pure ^{235}U and a detailed investigation into the non-equilibrium solution of the diffusion equation. Note that working out the stationary solution to the diffusion equation suffices to establish a *nuclear reactor* theory, as Heisenberg did in his G39 report. To do the same for a *nuclear explosive*, however, one needs to obtain the non-stationary solution, as Heisenberg did in his Farm Hall lecture in 1945. The accurate equation for the critical mass can also be derived, however, from the stationary solution. Furthermore, if Heisenberg, whose intuition was legendary, had made judicious assumptions as Frisch and Peirls did in March of 1940 on the Allied's side, should certainly have been able to obtain an estimate of only a few kilograms for the critical mass of an A-bomb.

129 [2], p. 170.
130 [2], p. 191.

3. In his lecture at Farm Hall on August 14 1945, Heisenberg started out with the correct equation for the critical radius of an atomic bomb, $R_c = \left(\frac{\pi}{2}\right) l$. Yet, surprisingly, instead of defining the diffusion length $\frac{D}{v} = (\text{diffusion length})^2 = l^2$ and derive from it the more detailed formula in terms of the MFPs for scattering and fissioning and the net neutrons produced per fission, Heisenberg introduced in his lecture on August 14, 1945, without justifying it, a "short cut" formula for l in which only the fission cross section or, equivalently, the MFP for fissioning, occurs. In other words, he expressed l only in terms of the most difficult to estimate fission parameter; the rest was contained in a constant of 6.2. The numerical values of the scattering parameter and net neutrons produced per fission Heisenberg quoted in his Farm Hall lecture lead to the conclusion that the acceptable range of the constant falls between 3.041 and 4.808; this implies that he recalled the wrong value of 6.2.

The consequence of Heisenberg's unexplained 6.2 factor in calculating critical mass may be as follows:

> Heisenberg's unusual approach in which only the fission cross section is the unknown used to calculate the critical radius (mass) of an atomic bomb indicates that Heisenberg may have employed this approach during the war -- and that he kept the theoretical derivation to himself, although the critical mass estimate of 10-100 kilograms appearing in an anonymous report in 1942 was, as concluded by Walker, probably obtained with Heisenberg's involvement.

Walker, in his book *German National Socialism and the Quest for Nuclear Power: 1939-1949*, expresses the view that[131] "*neither Army Ordnance officials nor the scientists were under great pressure. [...] It was easy to lose sight of the connection between nuclear fission research and warfare. After all, the war appeared almost won. By October of 1941, almost all of Europe was under German control.*" In other words, one could argue that after France's surrender on June 22, 1940, the German military might have concluded that the war could be won without employing A-bombs.

Heisenberg had all the knowledge necessary to develop the information, but he either did not do the necessary theorizing and calculation or, having done it, chose not to include the result in his reports. The fact that he did not provide such information, for whatever reason he may have had, failed to fulfill the first condition for building a nuclear explosive:[132] "*There was a strong initial drive by a small group of physicists to get the project off the ground*." We can safely conclude at least that due to this lack of more precise technical guidance with respect to atomic bombs, Heisenberg,

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131 [15], pp. 44-45.

132 Landsman (Ref. [10], p. 318) refers to Rhodes (Ref. [12]) and Bundy (Ref. [6]) when listing four conditions which must all be satisfied for an atomic bomb project to succeed: "(1) There was a strong initial drive by a small group of physicists to get the project off the ground; (2) From a certain point in time there was unconditional support from the Government; (3) Practically unlimited industrial resources and manpower were available; (4) There was an unprecedented concentration of brilliant scientists working on the project." If any one of these is not fulfilled, the project will fail. Thus the success of building an atomic bomb depends here on four factors. Any one of these factors, however, can cause failure of the project. Heisenberg's "inaction" as far as nuclear explosives are concerned, thus contributed, at the start of the war, to the failure to meet the first requirement stated above.

with his inaction, gave the German military no motivation to accelerate nuclear research and, consequently, any project the military might have considered for developing an atomic bomb. Although in 1942 the Germans determined that atomic bombs could not be finished before the war ended, a case can be made that the situation in December of 1940 when the G39 report -- which could have included information indicating that development of A-bombs might be feasible -- was issued, that building nuclear explosives during the war could not be ruled out. Note that at that time, before France's surrender on June 22, 1940, the outcome of the war was uncertain and the German military might indeed have decided to accelerate nuclear research.

Of course, Heisenberg's mathematical demonstration of feasibility might not have proven enough to trigger the initiation of a project. On the one hand, the German decision-makers might not have anticipated a larger war with their then-allied Russians and then-nonbelligerent Americans. But on the other hand, interest in an atomic bomb might have been sufficient to motivate them to take some action and make some level of increased commitment to such a project.

In hindsight it turns out to be the only time-frame in which an A-bomb project could have been started that might possibly have had a chance, no matter how remote, of succeeding.

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[3]Bernstein, Jeremy, "Heisenberg and the critical mass," Journal of Physics, 70 (9) 911-916 (2002).

[4]Blum, W., and Dürr, H.-P., and Rechenberg, H., *Werner Heisenberg's Gesammelte Werke / Collected Works, Series A/Part II, Original Scientific Papers / Wissenschaftliche Originalarbeiten*, Springer Verlag (1989).

[5]Bohr, Niels, "Resonance in uranium and thorium disintegrations and the phenomenon of nuclear fission," Physical Review 55(4) 418-419 (1939). Rhodes, in *The Making of the Atomic Bomb* (Ref. [12], p. 287), writes: "In January Bohr had produced a 700-word paper in three days to protect his European colleagues' priorities. Now, in his eagerness to spread the news of U235's special role in fission, he produced an 1,800-word paper in two days, mailing it to the Physical Review on February 7. 'Resonance in uranium and thorium disintegrations and the phenomenon of nuclear fission' was nevertheless written with care, more care than it received in the reading. Everyone understood that its basic hypothesis -- that U235, not U238, is responsible for slow-neutron fission in uranium -- though not everyone concurred without the confirmation of experiment."

[6]Bundy, McGeorge, *Danger and Survival: Choices About the Bomb in the first Fifty Years*, New York: Random House (1988).

Erratum

In the list of References on page 33 the reference

[15] Walker, Mark, *German National Socialism and the Quest for Nuclear Power: 1939-1049, Cambridge University Press (1989)*

was inadvertently omitted and should be re-inserted

(A piece of paper containing this correction was placed into each of the preprints which had already been completed briefly before the omission was detected)

[7]Cassidy, David C., *Uncertainty, the Life and Science of Werner Heisenberg*, W. H. Freeman & Company, NY (1992).

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[16]Walker, Mark, *Nazi Science: Myth, Truth, and the German Atomic Bomb*, Perseus Publishing (1995).

Acknowledgments

Since the conclusions reached in this paper are based on mathematical/scientific arguments, we had to exercise extraordinary care to avoid errors (for which we alone are responsible, if they should occur). We therefore took care to ensure that our equations are consistent with those shown in *The Los Alamos Primer*, a book that was used as a training manual by the Allies, as well as with some of the ideas expressed in Bernstein's publication in the "Journal of Physics" (2002), "Heisenberg and the critical mass."

We are grateful to the following people for helpful comments: Jeremy Bernstein, (the late) Harry Lustig, Helmut Rechenberg, Cameron Reed, Helmut Scherzer, and Ivan Todorov. But we wish to single out the contributions by Klaus Gottstein and Mark Walker whose innovative suggestions led to a more precisely worded paper.

Last, but not least, we are indebted to Stan and Nina Kurzban for making helpful suggestions and challenging all of our assertions and conclusions as well as for their editorial work, especially important due to the fact that the authors' native language is German, not English. (Mr. Kurzban is co-authoring a book with Meyer, one of the authors of this paper.)

Biographies

Carl H. Meyer, who holds a Ph.D. (1970) from the University of Pennsylvania, is best known as one of the designers of the IBM-developed algorithm that became the Data Encryption Standard (DES) in the U.S. in 1977 and a de facto standard in numerous other countries.[133] Dr. Meyer, an IEEE Fellow and the holder of 15 patents, gave numerous presentations on cryptography worldwide in seminars and congresses. His most significant publication is the book, co-authored with Dr. S. M. Matyas, *Cryptography - A New Dimension in Computer Data Security* (Wiley & Sons, 1982). This work, translated into Japanese, covers cryptographic algorithms and concepts as well as implementation details.

After retiring from IBM, Dr. Meyer pursued his interest in historical events of the last century. After seeing Frayn's drama "Copenhagen," which addresses Professor Werner Heisenberg's controversial Copenhagen trip in 1941 when he visited his mentor and friend Professor Niels Bohr, his curiosity was aroused. Consequently he started to write a book[134] in which he attempted to solve the riddle of Heisenberg's WW II involvement. Some of the major findings of this endeavor, with additional contributions from Dr. habil. Schwarz, are presented in this paper.

Günter Schwarz, a theoretical physicist, studied mathematics and physics at several German universities and received his doctoral degree in theoretical physics 1989 from the University of Karlsruhe. He has been a Lecturer at the University of Karlsruhe, Department of Physics (Oct. 1985 - Dez. 1998), an Assistant and Associate Professor, Department of Mathematics, University of Mannheim (Jan. 1989 - Oct. 1997), and a Research Fellow at the University of Calgary, Canada (Sept. 1992 - Oct. 1993, on a sabbatical). In June 1995 he habilitated and was appointed as Privatdozent (qualification to teach at German universities).

Dr. habil. Schwarz contributed to some 20 seminars and congresses world wide and authored about 25 publications in international journals in the field of theoretical physics (quantum field theory) and applied mathematics (differential equations, fluid mechanics) and actuarial sciences. He also published a book, "*Hodge decomposition - A method for solving boundary value problems"* (Springer 1995).

133 IBM, in the 1970's under the direction of Dr. W. L. Tuchman, was in the forefront of (commercial) cryptographic applications which up to that point was the domain of governments. One output was an algorithm which became a standard in the U.S. in 1977 under the name DES (Data Encryption Standard) - until it was replaced, due to advances in technology, in 2000 under the name AES (Advanced Encryption Standard).

134 The book is tentatively titled *The Puzzles of Heisenberg, and Why There Was No Nazi "Manhattan Project."*

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